A novel high order method for elliptic interface problems

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Major mathematical schemes for irregular interfaces and complex domains

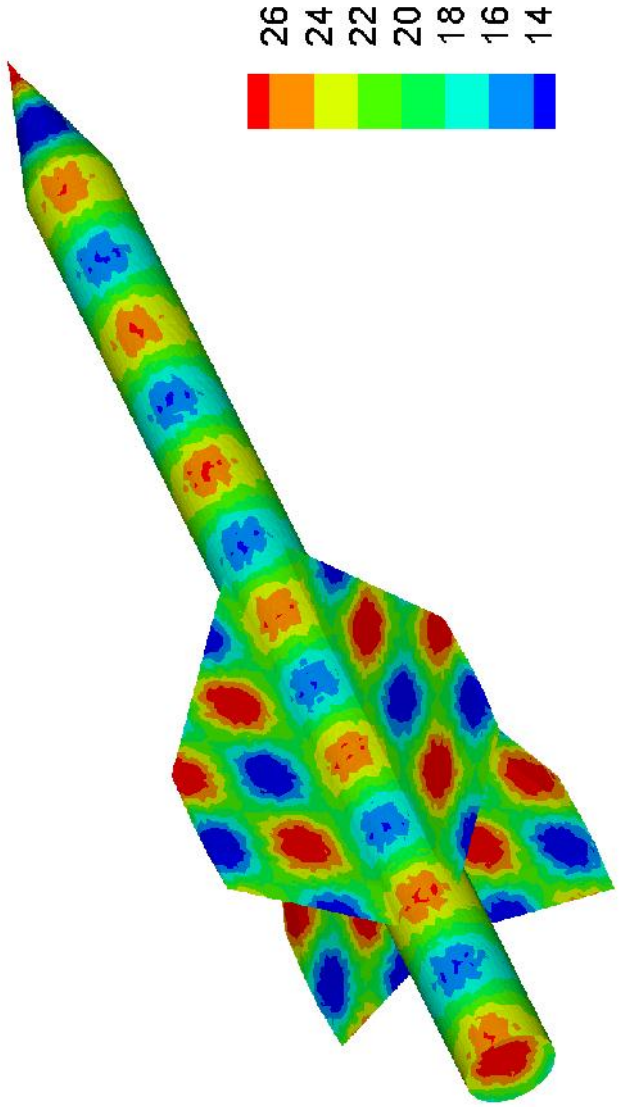
- Immersed boundary method (IBM, Peskin, 1977) (1st order)
- Immersed interface method (IIM, LeVeque and Li, 1994) (2^{nd} order)
- Ghost fluid method (GFM, Fedkiw and Osher, 2000) (1st order)

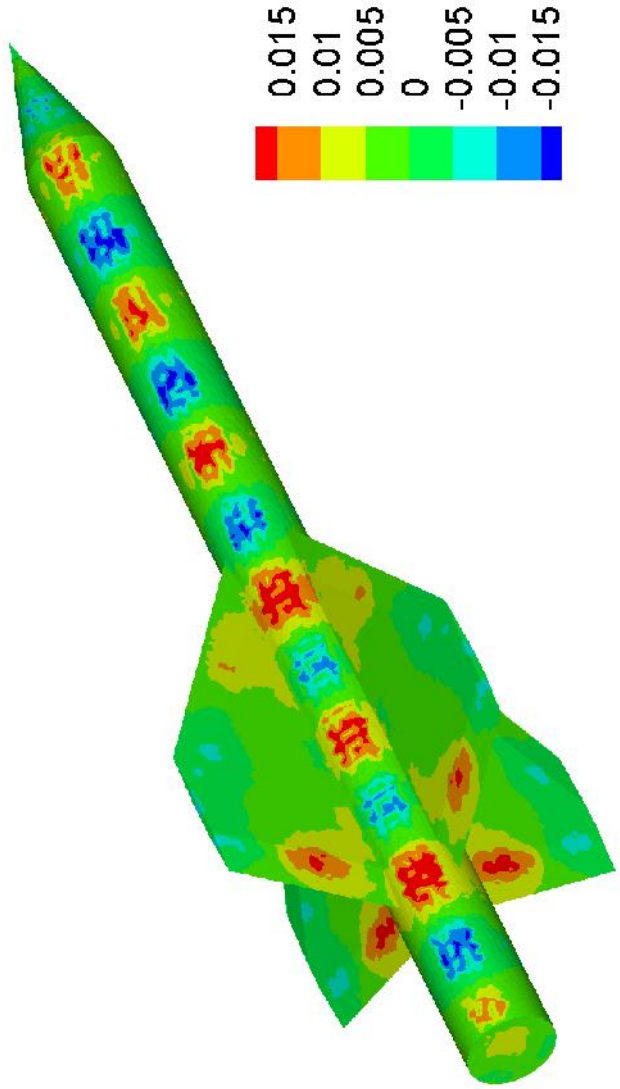
What do we do?

Solving the Poisson equation and the Helmholtz equation with interfaces

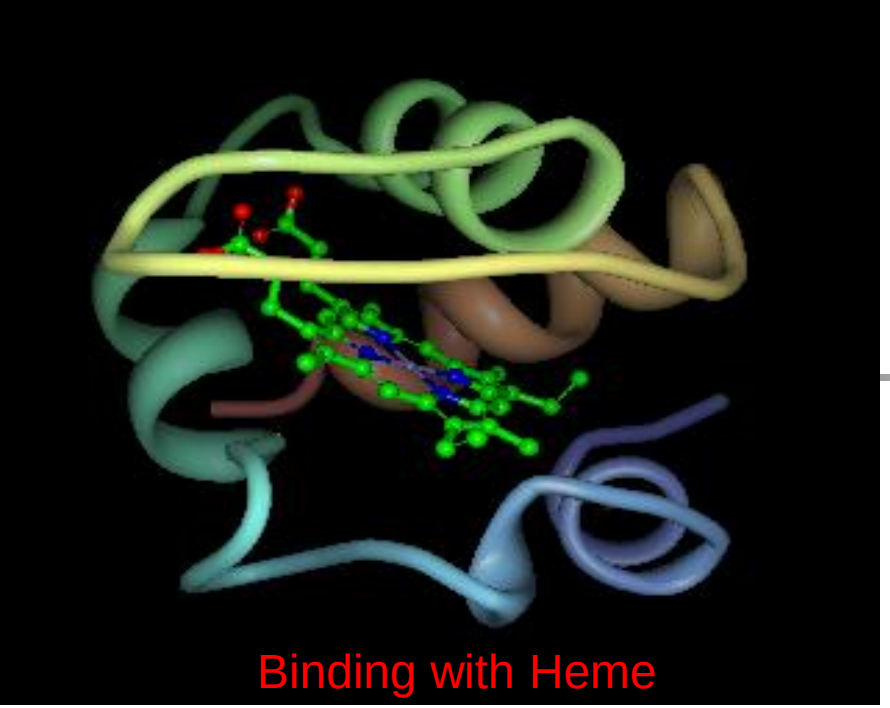
- Complex irregular 3D interfaces without grid generation
- The highest order accuracy available (4^{th}-16^{th} orders)
- The highest order accuracy for sharp-edged interfaces
- Efficient for interface and high frequency waves
- Efficient for multiply connected topology

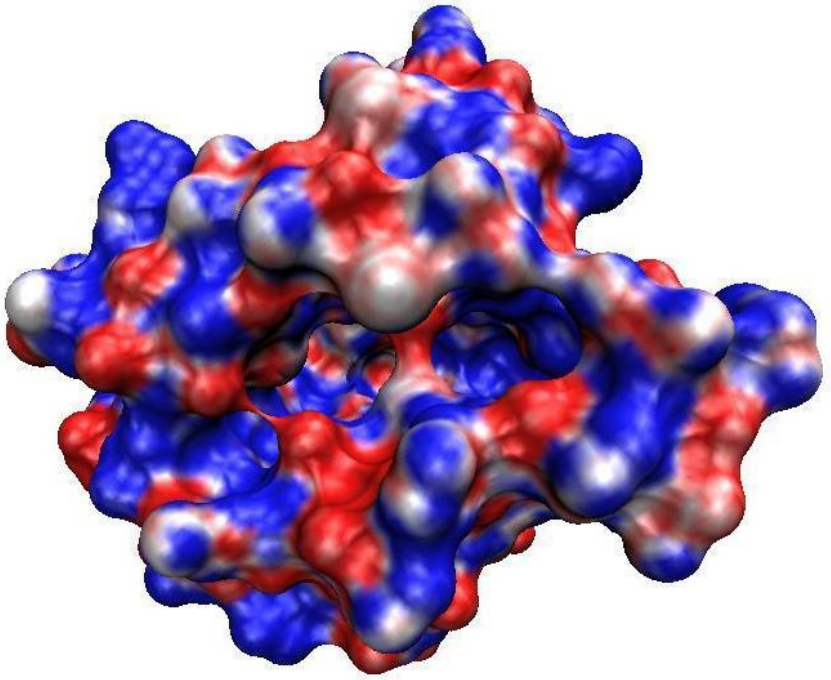
3D missile geometry



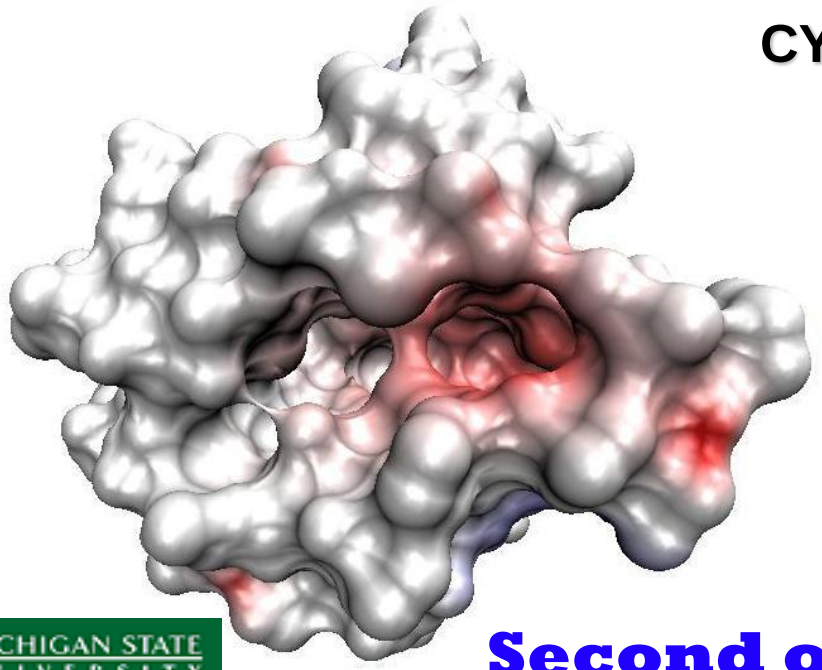


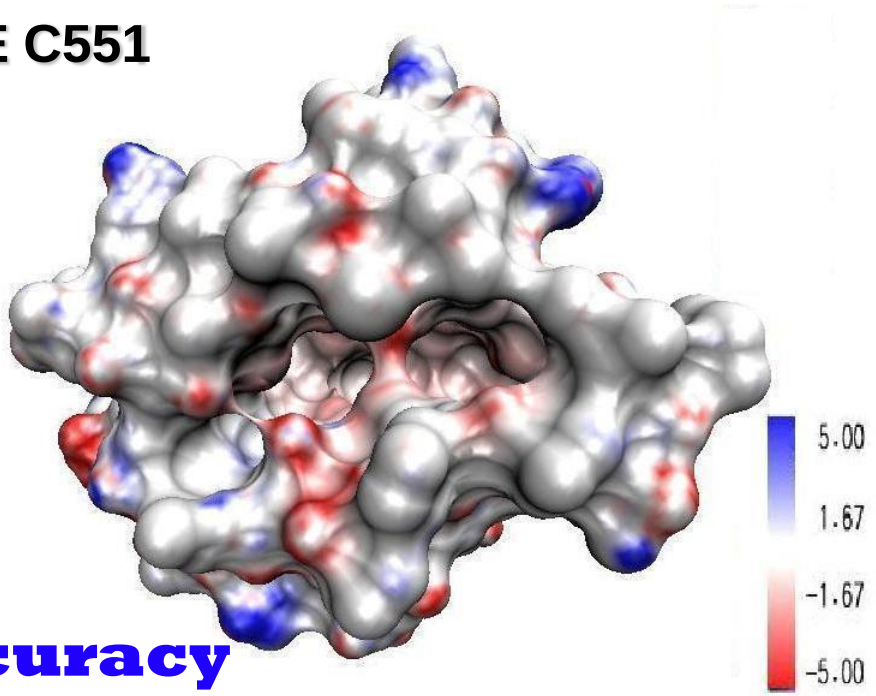
Second order accuracy





CYTOCHROME C551





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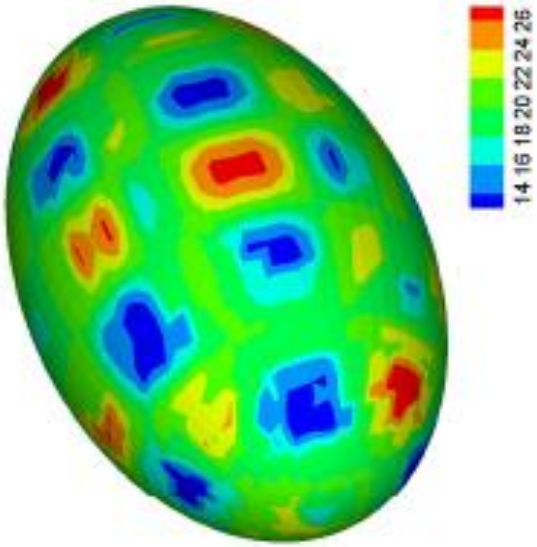
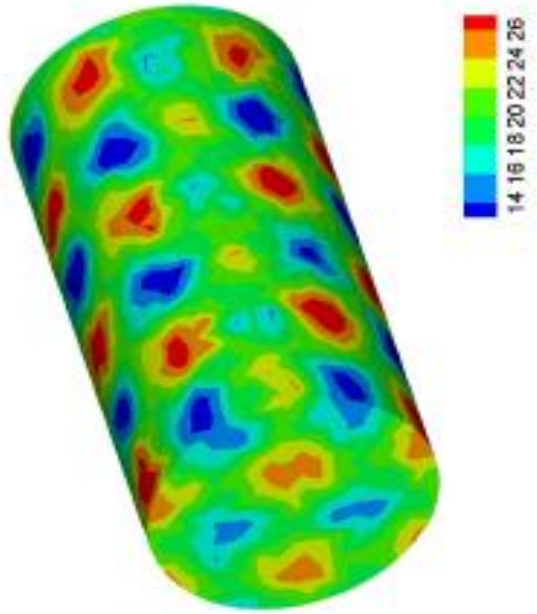
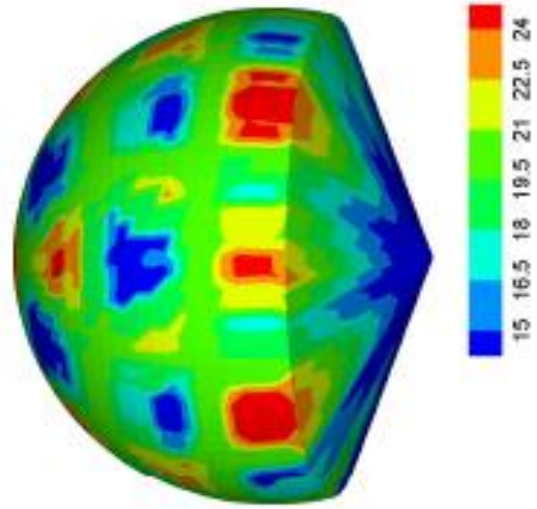
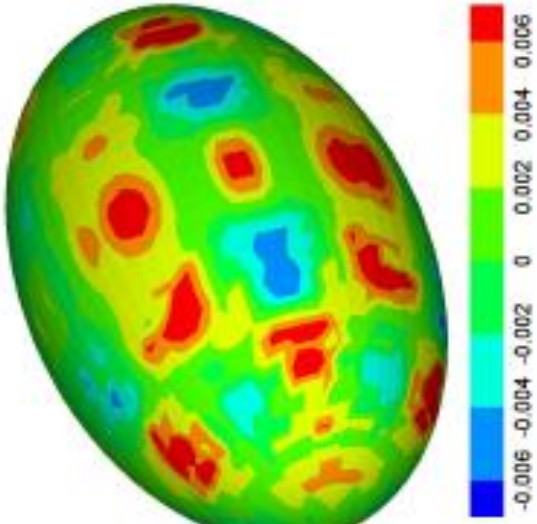
Second order accuracy

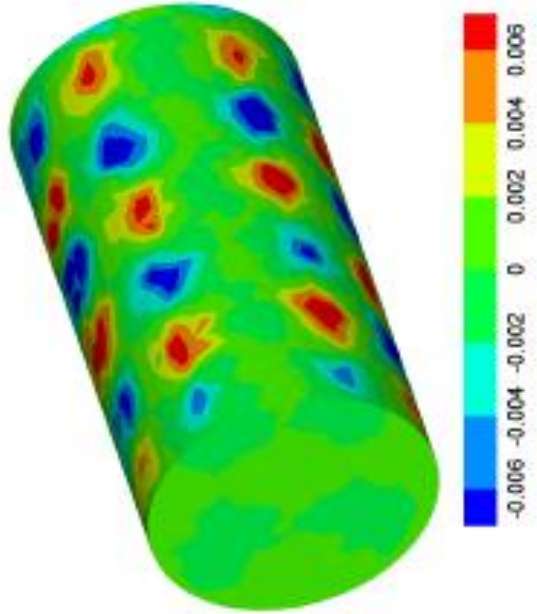


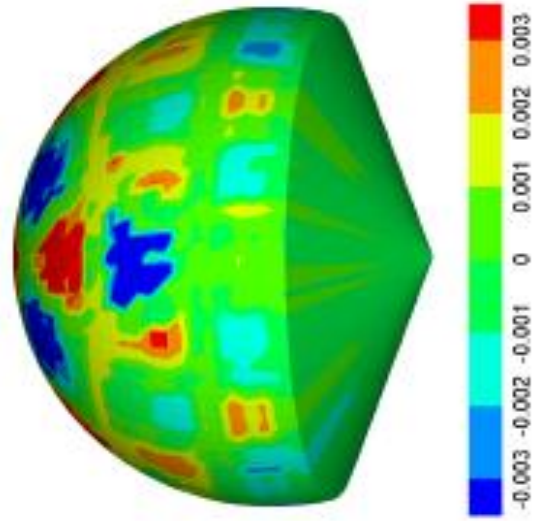




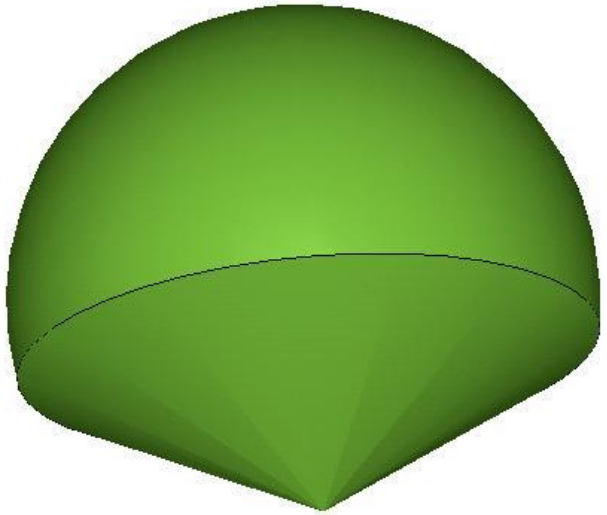
Fourth order accuracy

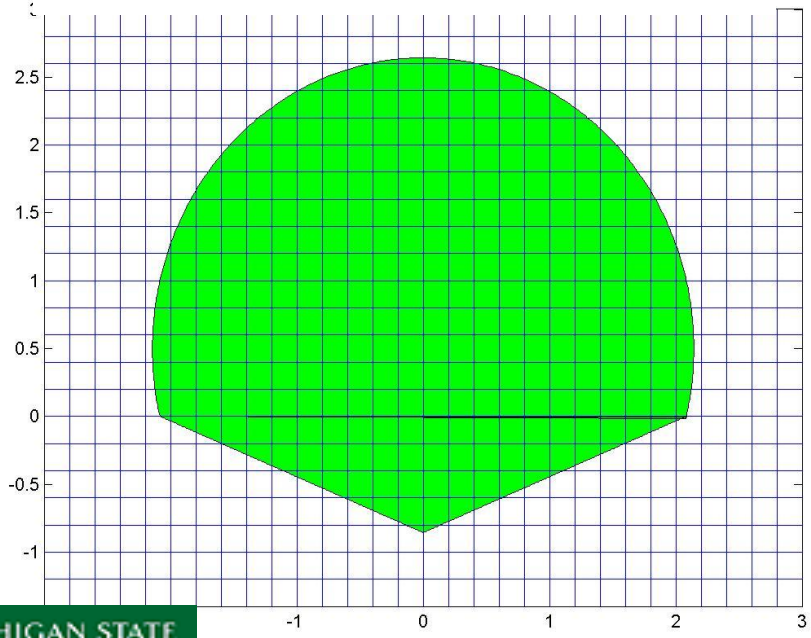






Matched interface & boundary (MIB)





Fourth order accuracy

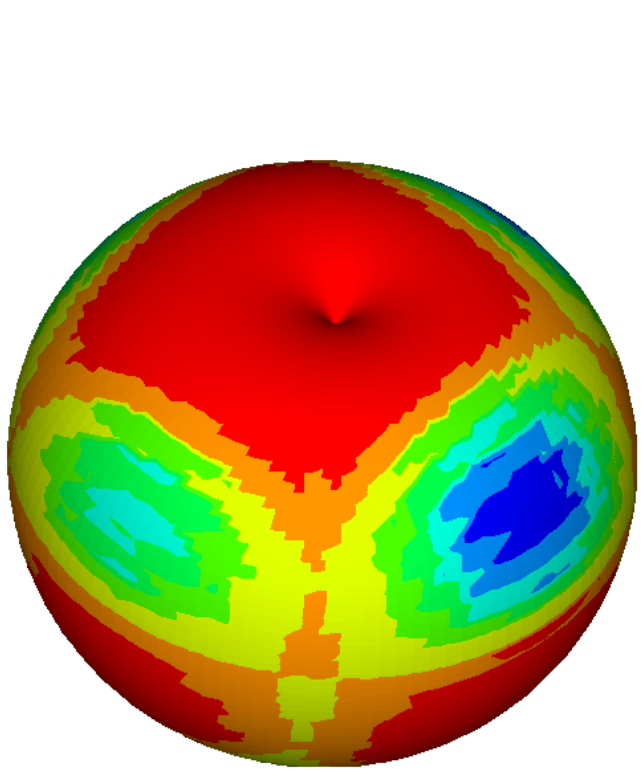
Mesh	Error	Order
0.2	7.09E-2	-
0.1	3.67E-3	4.27
0.05	2.00E-4	4.20

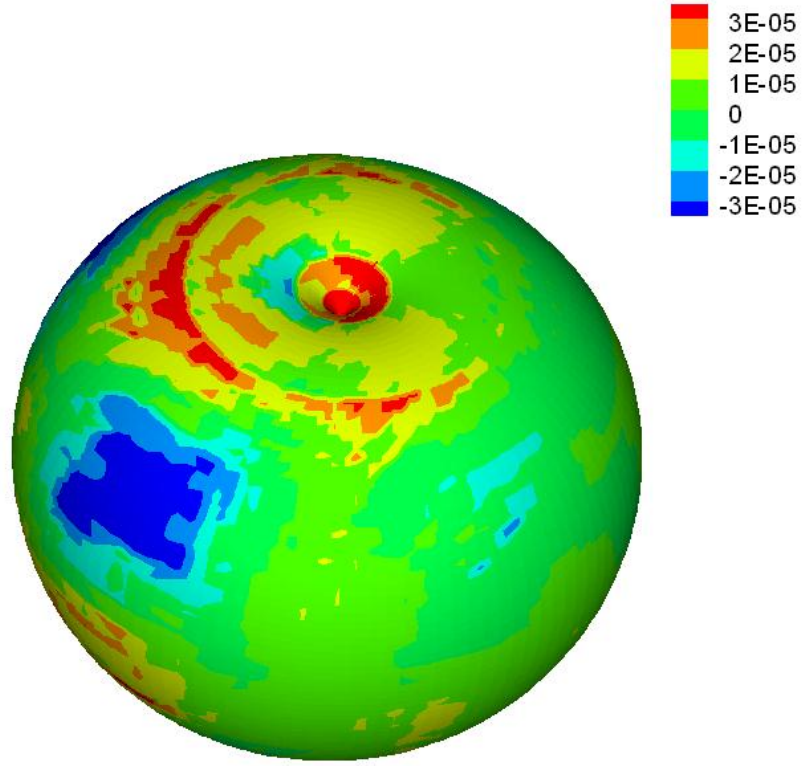
$$u^- = 10(x+y+z)+1$$

$$u^+ = 10\cos(kx)\cos(ky)\cos(kz)+20$$

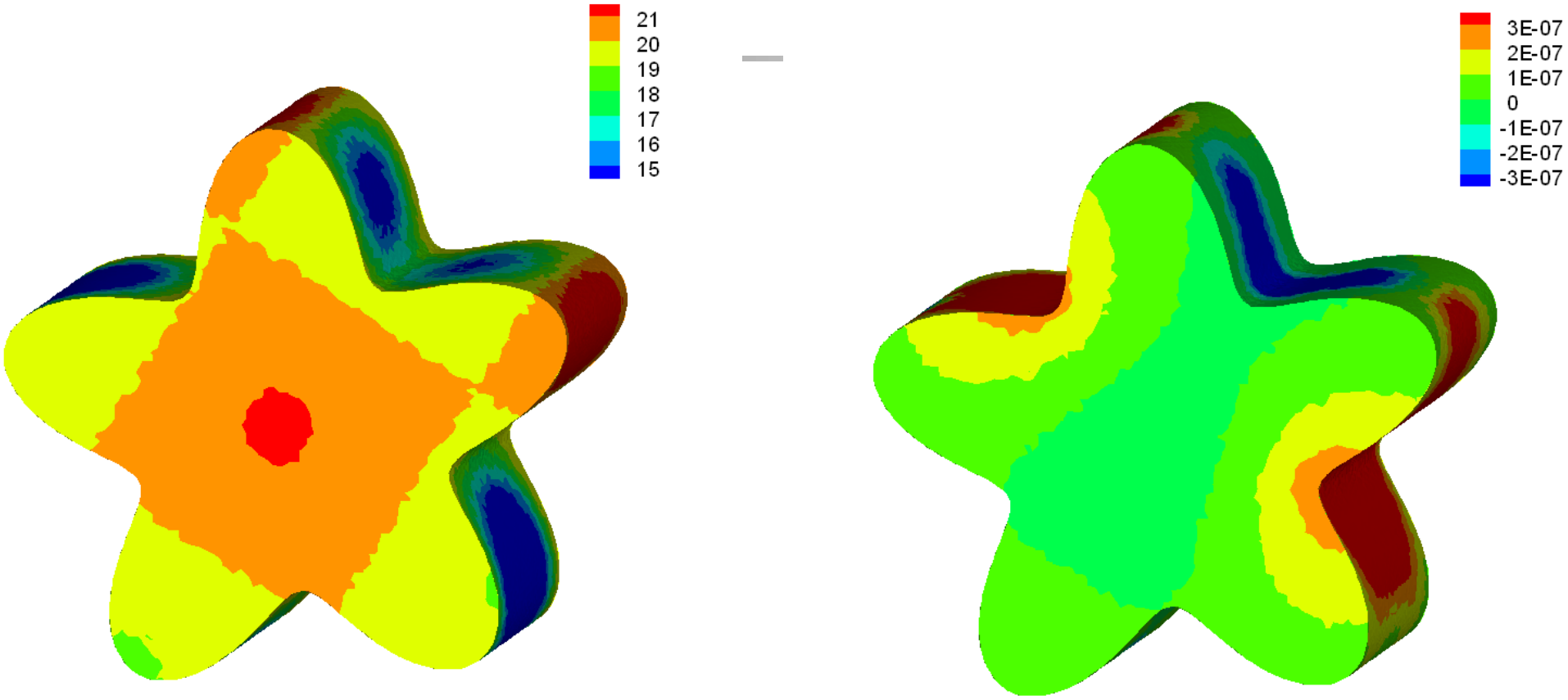
$$k=3, \quad \beta^- = z+15, \quad \beta^+ = (x+y)/2+10$$

Fourth order accuracy

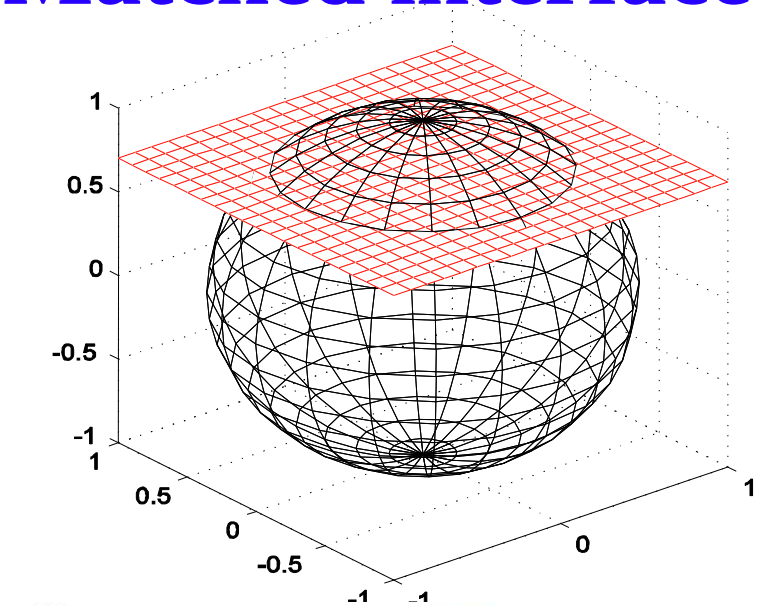


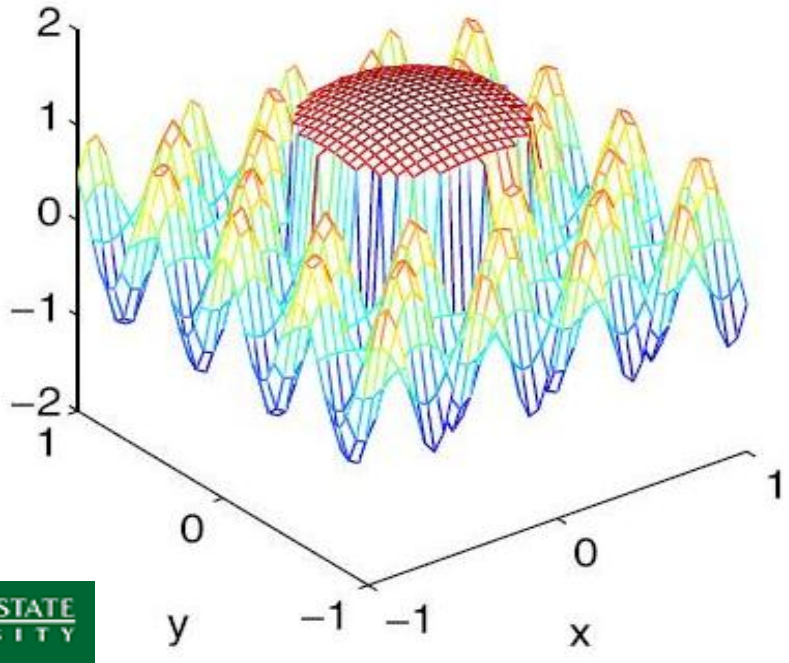


Fourth order accuracy



Matched interface & boundary (MIB)

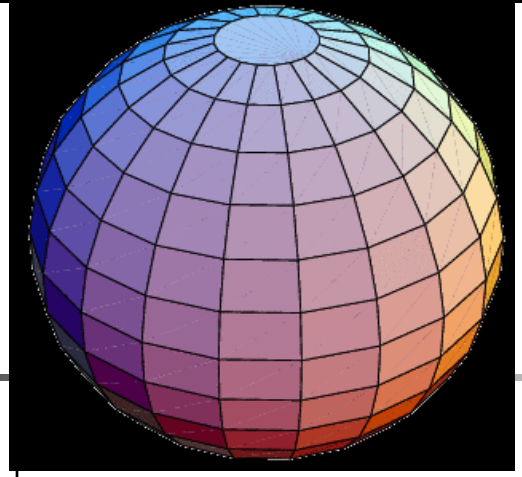


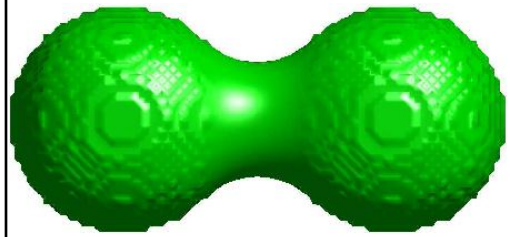


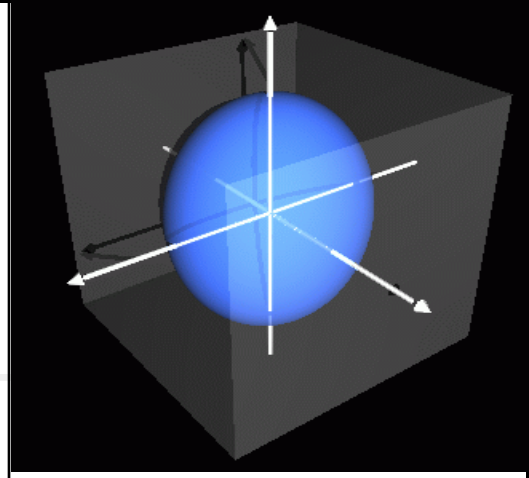
Sixth order accuracy

Mesh size	Error	Order
0.2	5.3E-7	-
0.1	5.4E-9	6.6
0.05	5.3E-11	6.7

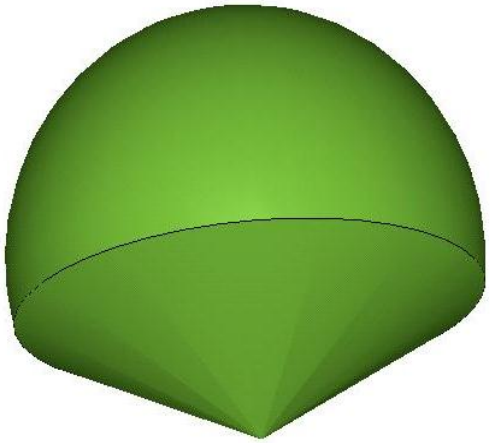
6th order accuracy

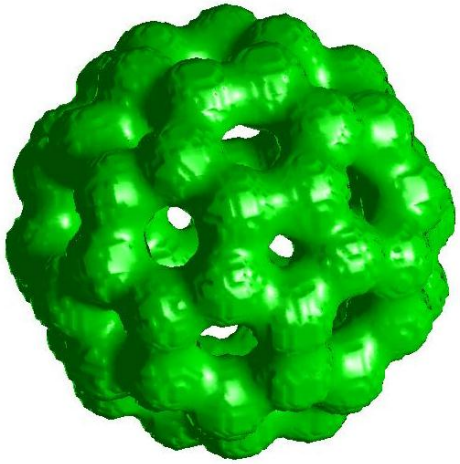


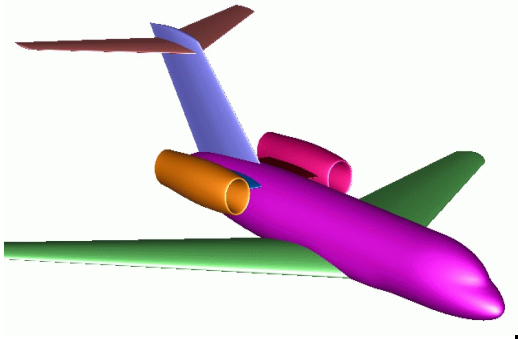




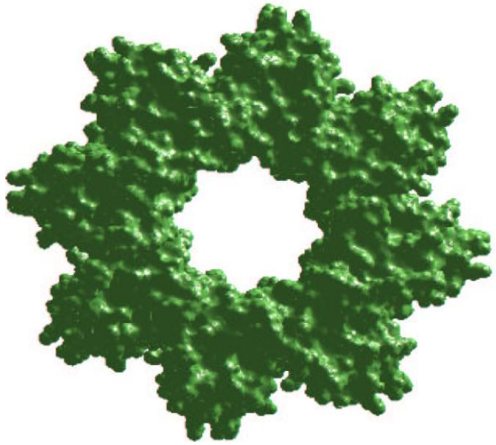
4th order accuracy

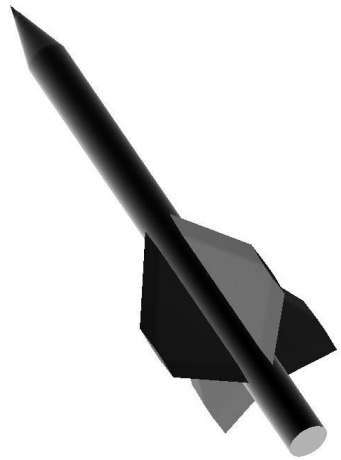


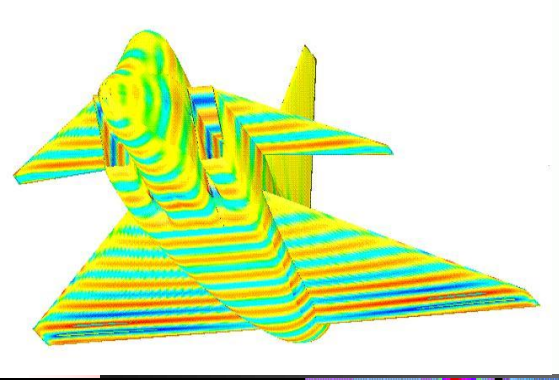


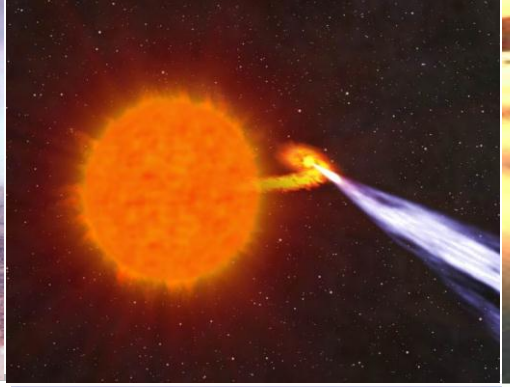


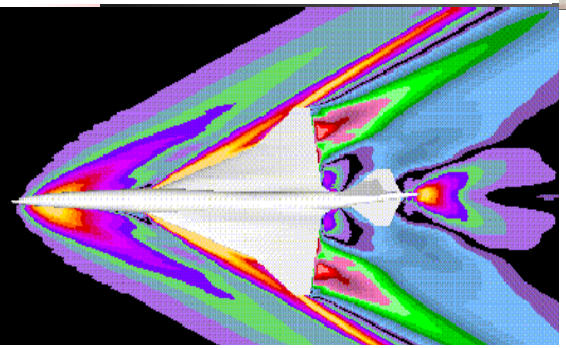
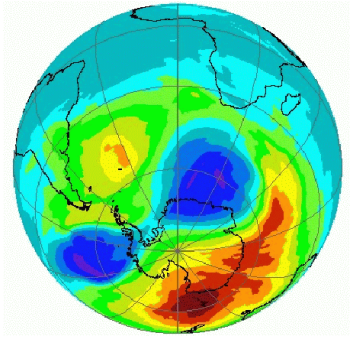
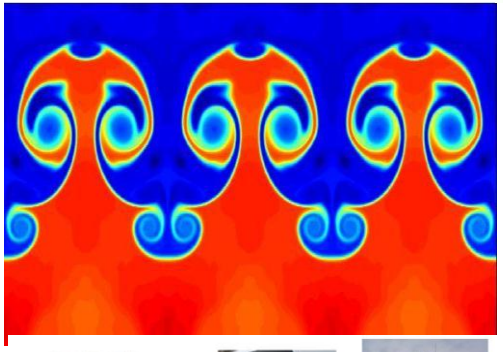
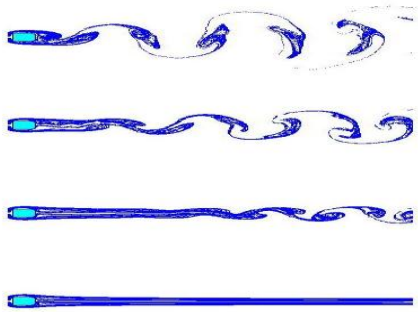
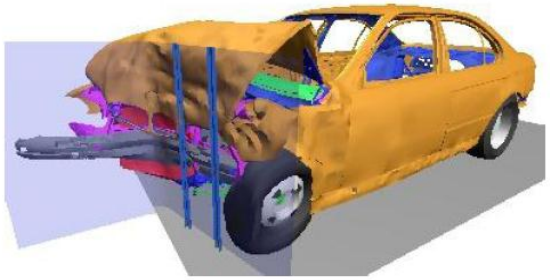
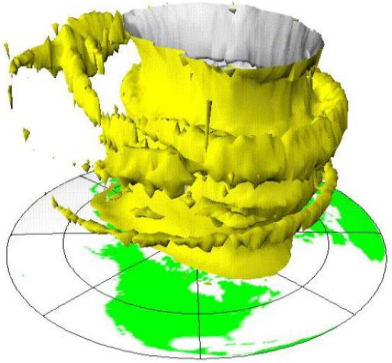
2nd order accuracy

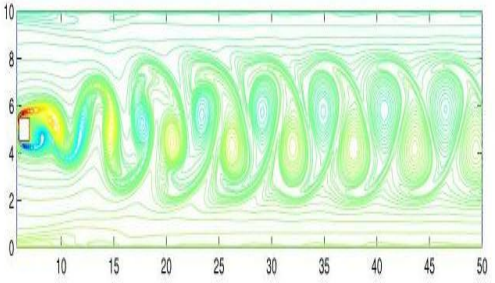








Radar
Wireless
Spectral
Efficiency

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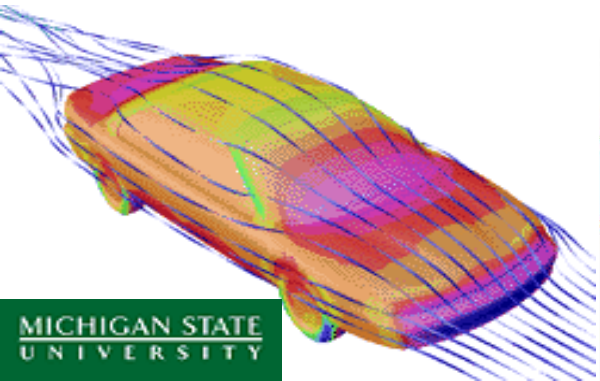
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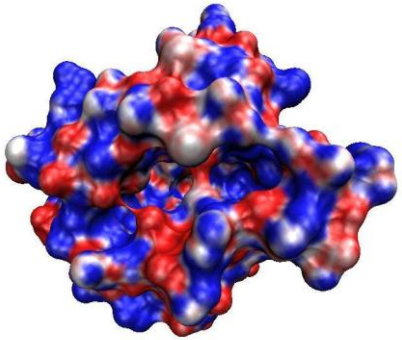
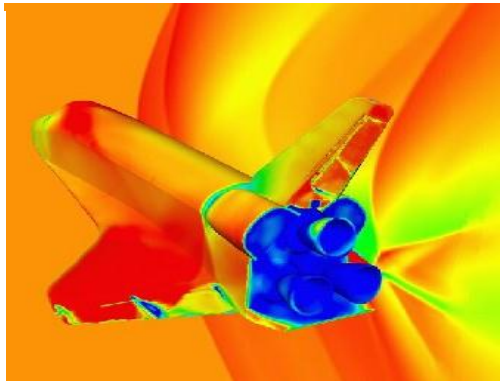
urban run off
landfills
leaking sewers
petrol station
industrial storage/ contaminated land
transpiration
evaporation
oil storage tanks
pesticides and fertiliser application
manure spreading
public water supply
uncovered road salt
septic tank
ploughing
Unsaturated zone
Saturated zone



References of our matched interface & boundary (MIB)

Maxwell's Equations (straight interfaces, 16th order, Zhao & Wei, JCP 2004)

2D Elliptic equations, (curved interfaces, 6th order, Zhou, Zhao, Feig & Wei, JCP 2006

2D Sharp-edged interfaces, (2nd order, Yu, Zhou & Wei, JCP 2006)

3D Sharp-edged interfaces, (2nd order, Yu, Geng & Wei, JCP 2007)

3D arbitrary interfaces (4th & 6th orders, Yu & Wei, JCP 2007)

3D singular charges & sharp-edged interfaces (2nd order, Geng, Yu & Wei, 2007)

Our team

April, 2006