**Math 153H** **Review for Midterm 2**

# 1 Volumes

1. Let $R$ denote the region bounded by the graph of $y = \sin(x)$ and the $x$-axis, for $0 \leq x \leq \pi$. Calculate the volume of the region obtained by rotating $R$ around the line $y = -1$.

2. Let $R$ be as in the previous problem. Calculate the volume of the region obtained by rotating $R$ around the line $x = -1$.

3. Repeat the previous two problems with the region $R$ replaced by the region bounded by the graph of $y = x - x^8$, and the $x$-axis, for $0 \leq x \leq 1$.

4. A cylindrical hole of radius 1 is drilled through the center of a ball of radius 8. What is the volume of the resulting solid? What is the volume of the material removed?

# 2 Work

1. Suppose a particle is moving on the $x$-axis, and that there is an ambient force given by $F = -\cos^5(x)\sin^3(x)$. (For example, the particle could be metallic and the force could be emitted from some crazy magnet.) How much work does the particle have to do for it to move from $x = 0$ to $x = \pi/2$. (Note that the force is negative in this region, so this means that the particle needs to do a *positive* amount of work to counteract the force.)

2. Consider the same particle as in the previous problem, with the same ambient force $F = -\cos^5(x)\sin^3(x)$. Assume a spring is the mechanism used to move the particle from $x = 0$ to $x = \pi/2$. The spring has one fixed end at $x = 0$. The particle is attached to the other end of the spring, and this other end has its natural length when $x = \pi/2$. It follows from Hooke's law that the force coming from the spring is $F_{\text{spring}} = k(\pi/2 - x)$, where $k$ is some constant depending on the spring. Determine all values of $k$ that ensure the particle makes it from 0 to $\pi/2$.

3. Suppose a spherical tank with radius 10 m is full of water. How much work is required to pump all of the water out of a pipe at the top, if the pipe is 1 m long?

4. Consider the same tank as in the previous problem, and assume you have just finished draining it completely. Now your ungrateful boss tells you to full it back up again. Assume the center of the tank is 12 m above the ground where the water is coming from. How much work is required to fill it back up?

## 3 Partial fractions

Evaluate the following.

1. $\int \frac{1}{x^2-a^2}\,dx$, where $a \in \mathbb{R}$. (Be careful about dividing by 0.)
2. $\int \frac{2x+1}{x^2+x+1}\,dx$
3. $\int \frac{1}{x^2+x+1}\,dx$
4. $\int \frac{2x+2}{x^2+x+1}\,dx$ (If you did the previous two, then this should be easy.)
5. $\int \frac{ax+b}{x^2+x+1}\,dx$, where $a \neq 0$. (Can you find an easy way to do this, given your answers to the previous 3 problems?)
6. $\int \frac{x^4+2x}{(x-1)^3}\,dx$
7. $\int \frac{x^3+1}{(x-1)(x-2)^2}\,dx$
8. $\int \frac{2x-1}{(x+1)(x^2+1)}\,dx$

## 4 Parametric equations/polar coordinates

1. Consider the parametric equations given by $(x(t), y(t)) = (t^4, 2t^8)$ for $1 \leq t \leq 16$. Describe this curve in terms of $xy$-coordinates.
2. Find 3 different sets of parametric equations that describe the curve from the previous exercise.
3. Let $C$ denote the circle of radius 4 with center $(6, 8)$. Describe $C$ using parametric equations $(x(t), y(t))$.
4. Repeat the previous problem, but require that $(x(0), y(0))$ is the lowest point of the circle and $(x(t), y(t))$ traverses the circle clockwise as $t$ increases.
5. Sketch the graph of $r(\theta) = \theta$ in polar coordinates, for $0 \leq \theta \leq 6\pi$.

## 5 Arclength/area

1. Compute the length of the curve described by $(x(t), y(t)) = (t^2, 3t^3)$ for $0 \leq t \leq 1$.
2. Compute the length of the curve described by $(x(t), y(t)) = (t^4, 2t^8)$ for $1 \leq t \leq 16$. *Hint: First do Problem 1 in the "Parametric equations/polar coordinates" section, above.*
3. Compute the length of $(x(t), y(t)) = (e^t \cos(t), e^t \sin(t))$ for $0 \leq t \leq \pi/2$.
4. Compute the length of the polar coordinate curve $r(\theta) = \theta$ for $0 \leq \theta \leq 2\pi$.

5. Compute the length of the polar coordinate curve $r(\theta) = \theta^2$ for $0 \le \theta \le 2\pi$.

6. Compute the area enclosed by the polar coordinate equation $r(\theta) = \frac{1}{2}\sin^2(\theta/2)$ for $\pi/4 \le \theta \le 3\pi/4$.

# 6 Limits

Use the definition of limit to prove the following statements.

1. $\lim_{x\to a} mx + b = ma + b$, where $m, a, b \in \mathbb{R}$.

2. $\lim_{x\to 3} 1/x = 1/3$.

3. $\lim_{x\to 0} x^n = 0$, where $n$ is 18 times the day of the month on which you were born.