Math 153H **Recitation Worksheet #5** **Oct 7, 2014**

Problem 1. Let A be a positive number, and consider the parametric equations

$$x(t) = t^3 - t, \qquad y(t) = \frac{A}{1+t^2}.$$

Let C denote the image in the xy-plane.

(a) Determine where (if at all) the curve C intersects the x- and y-axes.

(b) Use calculus to find the point(s) on C with the highest y-value.

(c) Does C have a point with the lowest y-value?

(d) Find the points on C with vertical tangent lines.

(e) The curve C has exactly one point of self-intersection. That is, there are two times distinct t_0, t_1 for which

$$(x(t_0), y(t_0)) = (x(t_1), y(t_1)).$$

Find the times t_0, t_1 and the associated point in the xy-plane.

(f) Since the curve C intersects itself at a point, it has two tangent lines at that point. There is exactly one value of A for which these tangent lines are perpendicular. Find this value of A.

(g) Show that C is symmetric about the line $x = 0$ in the xy-plane.

(h) Try to sketch C.

Problem 2. The parametric equations

$$x(t) = \sqrt{3} sin(t) + \frac{1}{2}\cos(t), \qquad y(t) = -\sin(t) + \frac{\sqrt{3}}{2}\cos(t)$$

describe a tilted ellipse. Let B be a box in the plane with sides parallel to the x- and y-axes. Suppose B is chosen perfectly so that the sides of B are tangent to the ellipse. Find the dimensions of B.

Problem 3. Let $A = \{1, 2, 3\}$ and $B = \{\Box, *\}$.

(a) Define a function $f : A \to B$ by its graph

$$\{(1, \Box), (2, \Box), (3, *)\} \subseteq A \times B.$$

Determine $f(1), f(2)$ and $f(3)$. Try to sketch $A \times B$, and indicate the graph of f.

(b) Determine which of the following sets are graphs of functions from A to B, and which are not.

$$\{(1, \Box), (3, *), (2, \Box)\}$$
$$\{(3, *), (2, \Box), (\Box, 2)\}$$
$$\{(2, *), (1, *), (2, \Box), (3, *)\}$$

(c) There are a total of 8 functions from A to B. Find these; you may describe each function by its graph, if you want.

(d) How many functions are there from A to A?