

## Math 396 - Quiz 2 Study Guide

### 1. (Fundamental Theorem of Arithmetic)

(a) Prove that if  $n$  is an integer greater than 1, then there are prime numbers  $p_1, p_2, \dots, p_k$  so that

$$n = p_1 p_2 \dots p_k.$$

(b) State the fundamental theorem of arithmetic. (The result from part (a) is only part of the fundamental theorem.)

(c) Explain why the integer 1 is not considered a prime.

2. Let  $a, b, c$  be real numbers, with  $a \neq 0$ , and consider the polynomial function  $f(x) = ax^2 + bx + c$ .

(a) Find real numbers  $r, h, k$  such that

$$f(x) = r(x - h)^2 + k.$$

You should express  $r, h, k$  in terms of  $a, b, c$ .

(b) Find numbers  $\alpha, \beta_1, \beta_2$  such that

$$f(x) = \alpha(x - \beta_1)(x - \beta_2).$$

You should express  $\alpha, \beta_1, \beta_2$  in terms of  $a, b, c$ .

3. Determine whether the following statement is true, and prove your answer:

Suppose  $n, a, b$  are positive integers and  $n$  divides  $ab$ .

Then  $n$  divides  $a$  or  $n$  divides  $b$ .

4. Restate each of the following sentences by explicitly writing all quantifiers involved as  $\forall$  or  $\exists$ . That is, all variables should have a quantifier ( $\forall$  or  $\exists$ ), as well as a domain (e.g.,  $\mathbb{R}$  or  $\mathbb{N}$ ) indicating the set to which the variable belongs. Finally, determine whether your statement is true or false.

(a) *Every linear polynomial  $mx + b$  has a root.*

(b) *Every quadratic  $ax^2 + bx + c$  has a root.*

(c) *There is no largest prime number.*

5. (a) Define what it means for two functions to be equal.

(b) Let  $a_1, b_1, c_1, a_2, b_2, c_2$  be real numbers, and consider the functions  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  defined by

$$f(x) = a_1x^2 + b_1x + c_1, \quad g(x) = a_2x^2 + b_2x + c_2.$$

Show that  $f = g$  if and only if

$$a_1 = a_2, \quad b_1 = b_2, \quad \text{and} \quad c_1 = c_2.$$

6. (a) Find all real solutions  $y$  to the equation  $y - \sqrt{y} - 12 = 0$ .

(b) Find all complex solutions  $y$  to the equation  $y^4 - 2y^2 + 7 = 0$ .

7. (Division base 3) Let  $n \in \mathbb{N}$ . We say a sequence of numbers

$$\dots a_3a_2a_1a_0$$

is the **ternary expansion** of  $n$  if  $0 \leq a_k < 3$  for all  $k \geq 0$ , and if

$$n = \sum_{k=0}^{\infty} a_k 3^k.$$

The ternary expansion of a natural number is unique (you do not need to prove this). Also,  $a_k = 0$  for all but finitely many  $k$  (you do not need to prove this), and so we do not usually write these values.

(a) Show that the ternary expansion of 17 (this is written in decimal notation) is

$$122$$

(b) Find the ternary expansion of 44 (this is written in decimal notation).

(c) Let  $a, b$  be positive integers. The division algorithm says there are unique integers  $q, r$  such that

$$a = qb + r, \quad 0 \leq r < b.$$

Describe an algorithm for finding  $q$  and  $r$  assuming you are given the ternary expansions of  $a$  and  $b$ . (Think 'long division', but with base 3 instead of base 10.)

(d) Use your algorithm from (c) to find  $q, r$  in the case where  $a = 17$  and  $b = 44$ . (First convert  $a, b$  to ternary, and then run your algorithm. You may give  $q, r$  in ternary, if you want.)