Homework due October 11

1. Let $G = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is a bijective function}\}$. Show that the set G together with function composition satisfies the four group axioms. That is,

   A_1. **Closure:** $\forall f, g \in G,\ f \circ g \in G$.
   Is this statement true? Give a brief explanation - you can use previous knowledge. You do not need to provide a detailed proof.

   A_2. **Associativity:** $\forall f, g, h \in G,\ (f \circ g) \circ h = f \circ (g \circ h)$
   Is this statement true for functions? No proof is needed.

   A_3. **Identity element:** $\exists e \in G$ such that $\forall f \in G,\ f \circ e = e \circ f = f$.
   What function corresponds to the identity element? Does it belong to the set G?

   A_4. **Inverse element:** $\forall f \in G, \exists g \in G$ such that $f \circ g = g \circ f = e$.
   What function corresponds to the inverse element of f? Does it belong to the set G? Why?

   Is it true that G together with function composition satisfies the following additional property:
   Commutativity: $\forall f, g \in G, f \circ g = g \circ f$?
   Support your answer by an example.

2. Consider the following axioms.

   Undefined terms: Gub, rok

   A_1. There exist exactly five gubs.

   A_2. Each rok is a subset of those five gubs.

   A_3. There exist exactly two roks.

   A_4. Each rok contains at least two gubs.

Which of the following models satisfy the above set of axioms? If a model does not satisfy the axiomatic system, explain which of the axioms is violated.

Model 1. The set of gubs is given by $\{A, B, C, D, E\}$. The set of roks is given by $\{\{A, B\}, \{B, C, D\}\}$

Model 2. The set of gubs is given by $\{1, 2, 3, 4, 5\}$. The set of roks is given by $\{\{1, 2\}, \{2, 3, 4, 5\}\}$

Model 3. The set of gubs is given by $\{\clubsuit, \diamondsuit, \heartsuit, \spadesuit, \triangle\}$. The set of roks is given by $\{\{\clubsuit\}, \{\diamondsuit, \heartsuit, \spadesuit\}\}$

Model 4 Consider the set of gubs and roks given by the figure below (gubs are small labeled circles, and roks are large ellipses). Which of the above models is it equivalent to? Write a one-to-one correspondence between the elements of the two equivalent models.