

Give a careful proof of the following, following the steps below.

PROPOSITION: Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be two injective functions. Then the composite function $g \circ f : A \rightarrow C$ is also injective.

Proof: Suppose $f : A \rightarrow B$ and $g : B \rightarrow C$ are injective. To prove the composition $g \circ f : A \rightarrow C$ is injective, we assume

$$g(f(a_1)) = g(f(a_2))$$

for some $a_1, a_2 \in A$. We need to show that $a_1 = a_2$. Since g is injective, it follows that

$$f(a_1) = f(a_2).$$

Since f is injective, it follows that $a_1 = a_2$, as desired. Q.E.D.