23.7 (ii) We will to prove by contradiction that $\sqrt{5}$ is irrational. So we assume that

$$\sqrt{5} = p/q \tag{1}$$

where $p, q \in \mathbb{Z}$ are integers, and $q \neq 0$. By canceling common factors, we may suppose that p and q have no prime factors in common. Squaring both sides of (1), and multiplying by q^2, gives

$$5q^2 = p^2. \tag{2}$$

This shows that 5 divides p^2. Since 5 is prime, it must be the case that 5 divides p. So we can write

$$p = 5k$$

for some $k \in \mathbb{Z}$. Substituting this in (2) gives

$$5q^2 = 25k^2,$$

and hence

$$q^2 = 5k^2.$$

This shows that q^2 is divisible by 5. Just as before, this implies that q is divisible by 5. So we have that p and q are both divisible by 5. This contradicts the statement that p and q have no common factors.

□

23.7 (iii) If we tried to repeat the above proof with $\sqrt{4}$ in place of $\sqrt{5}$, the argument would fail. This is because 5 is prime, but 4 is not. For example, it is possible for 4 to divide p^2, but for 4 not to divide p.