28.6 (iii): Suppose $x = qy+r$, where $x, y, q, r \in \mathbb{Z}$ (you do not need to assume that y does not divide x). We want to prove that $gcd(x, y) = gcd(y, r)$. Let $d := gcd(x, y)$. This means that d divides x and y. By the relation $r = x - qy$, it follows from Theorem 27.5 that d divides r. So d is a common divisor of y and r, and hence $d \leq gcd(y, r)$.

Next, we want to prove that d is the greatest integer that divides y and r. For this, we follow Houston's suggestion to prove by contradiction (however, there is a direct approach which also works). Then we assume there is some e which divides y and r, and $d < e$. Then e also divides $x = qy + r$, and so $e \leq gcd(x, y) = d$, which is a contradiction.

Here is an alternative way to finish the proof after the first paragraph: By definition, $gcd(y, r)$ divides y and r, so it must divide $x = qy + r$ by Theorem 27.5. This implies $gcd(y, r) \leq d = gcd(x, y)$, which proves $gcd(x, y) = gcd(y, r)$.

28.19 (i): $gcd(14592, 6468) = 12$ and $gcd(-12870, 4914) = 234$.