

1. Compute the sup, inf, max and min (whenever these exist) for the following sets.

a. $S_1 = \left\{ 1 + \frac{1}{n} \mid n \in \mathbb{N} \setminus \{0\} \right\}$

b. $S_2 = (-3, -1] \cup [1, 2) \cup \{7\}$

c. $S_3 = (-3, -1] \cup [1, 2) \cup \{-4\}$

d. $S_4 = \{y \mid y = x^2 - 9, \text{ and } x \in \mathbb{R}\}$

e. $S_5 = \{x \mid x^2 - 9 < 0, \text{ and } x \in \mathbb{R}\}$

2. Is it possible for a subset of \mathbb{R} to have a maximum, but no supremum? If yes, give an example. If no, provide a brief justification.

3. Prove Beck Proposition 8.43.

4. Prove Beck Proposition 8.50.

5. Prove Beck Proposition 8.45. *Hint: Use Proposition 8.50 twice.*