1. Consider the following axioms regarding inequalities.

   A1 For all real numbers a, b, c, if $a \leq b$ and $b \leq c$ then $a \leq c$.

   A2 For all real numbers a, b, c, if $a \leq b$ then $a + c \leq b + c$.

   A3 For all real numbers a, b, c, if $a \leq b$ and $0 \leq c$ then $ac \leq bc$.

   Prove the statements below. Do not assume anything else about *inequality* $\leq$ except that which appears in A1, A2 and A3. However, you may use any basic facts about *equality* $=$ that you wish.

   (a) For all real numbers a, b, if $0 \leq a$ and $a \leq b$ then $a^2 \leq b^2$.

   (b) For all real numbers a, if $a \leq 0$ then $0 \leq -a$.

   (c) For all real numbers a, b, if $b \leq a$ and $a \leq 0$, then $a^2 \leq b^2$.

   (d) For all real numbers b, $0 \leq b^2$.

   (e) For all real numbers a, b, $ab \leq \frac{1}{2}(a^2 + b^2)$. *Hint: Consider* $(a - b)^2$.

   (f) For all real numbers a, b, δ, if $\delta \neq 0$ then $ab \leq \frac{1}{2}(\delta^2 a^2 + \delta^{-2} b^2)$.

   (g) For all real numbers a, b, $ab = \frac{1}{2}(a^2 + b^2)$ if and only if $a = b$.

   (h) For all non-negative real numbers a, b, $\sqrt{ab} \leq \frac{1}{2}(a + b)$. This is called the *arithmetic-geometric mean inequality*.

2. Exercise 3.50 (3.36) from the book.

3. For $x \in \mathbb{R}$, define $|x|$ to be x if $x \geq 0$ and $-x$ if $x < 0$. Using this definition of the absolute value, prove that if $x + y < 0$, then $|x + y| \leq |x| + |y|$.