Take-Home Quiz II

MTH 132
Fall Semester 2008
Date: Nov 19, **Due: Nov 24**

PID: ____________________
NAME: ____________________
section: $2and15$

Chapter 3

0.1 The definition of the derivative

1. The derivative of $f(x)$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

or

$$f'(x) = \lim_{z \to x} \frac{f(x) - f(z)}{x - z}.$$

(a) Use the definition to find the derivative of $y = \sqrt{x}$ for $x > 0$.

(b) Use the definition to show that $f(x)$ is differentiable at $x = 0$, provided that

$$f(x) = x^2 \sin \frac{1}{x} \text{ for } x \neq 0 \text{ and } f(0) = 0.$$

(c) Explain the reason why $|x - 2|$ is not differentiable at $x = 2$.

2. For what value or values of constant m, if any, is

$$f(x) = \sin 2x, \;\; x \leq 0$$

$$f(x) = mx, \;\; x \geq 0$$

(a) continuous at $x = 0$.

(b) differentiable at $x = 0$.

0.2 The tangential line

3. Find the line whose slope is 3, and which is also through $(1, 2)$.

$*$ We know three method to express a given curves as follows:

(a) $y = f(x)$ (explicit). ex) $y = x^2 + x + 1$.

(b) $F(x, y) = c$ (implicit). ex) $x^2 + y^2 = 2x + 1$

(c) Parametric equations. ex) $x = \sin t$ and $y = \cos t$.

So, each expression has own method to get the slope and the tangential line.
4. Let $y = x \sin(2x + \frac{\pi}{8})$.

(a) Find the derivative.

(b) Find the tangential line at $x = \frac{\pi}{16}$

(c) Find the line perpendicular to the tangential line at $x = \frac{\pi}{16}$ above.

5. Find equations for the horizontal tangents to $y = x^3 - 3x - 2$.

6. We consider the curve $x^2 + y^2 = y^4 - 2x$ expressed implicitly.

(a) Find its slope at (-2,1) and the tangential line at the same point.

(b) Find the second derivative $\frac{d^2y}{dx^2}$.

7.

(a) What is the curve $\frac{x^2}{4} + \frac{y^2}{9} = 1$? Moreover, find the area of the inside region of the curve.

(b) Find $\frac{dy}{dx}$ of $x = 2\cos 2t$ and $y = 3\sin 2t$ at $t = \frac{\pi}{6}$ and the tangential line at the same point.

(c) Find $\frac{d^2y}{dx^2}$

8. We have two parametric equations $(\cos t, \sin t)$ and $(\cos 2t, \sin 2t)$. Both of them are on the unit circle $x^2 + y^2 = 1$. Explain the difference between them from the viewpoint of the movement of particles along time t.

0.3 Related rate

Speaking of the related rate, we want to find $\frac{dA}{dt}$. But, we have the relation $A = A(x)$ between A and another variable x, and its derivative $\frac{dx}{dt}$. Therefore, we can get $\frac{dA}{dt}$ by the chain rule

$$\frac{dA}{dt} = \frac{dA}{dx} \cdot \frac{dx}{dt}.$$

It is very important to establish the relation equation $A(x)$ from problem, since the derivative $\frac{dx}{dt}$ is usually given.

9. (Distance) A balloon is rising vertically above a level, straight road a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it. Now, we want to know how fast the distance $s(t)$ between the bicycle and balloon is increasing 3 sec later.

(a) Find the relation among $s(t)$, $x(t)$ and $y(t)$, i.e., express $s(t)$ by $x(t)$ and $y(t)$.

(b) What are $x'(t)$ and $y'(t)$ from the problem ?

(c) What is $s'(t)$ at $t = 3$?

10. (Distance) A 13-ft ladder is leaning against a house when its base stats to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.

(a) Find the relation between $x(t)$ and $y(t)$.

(b) What are $x'(t)$ from the problem ?

(c) What is $y'(t)$ at $t = 1$?

11. (Area) Suppose that A is the area of the a circle with radius r. Write A in terms of r, and also find the equation that relates $\frac{dA}{dt}$ to $\frac{dr}{dt}$.

12. (Volume) Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of $10in^3$/min.

(a) Find the relation between the volume V_1 of coffee in conical filter and the deep $d(t)$ of cone. (Hint: refer to Prob 4 at pp218)

(b) What is $V_1'(t)$ at $d = 5$?

(c) Find the relation between the volume V_2 of coffeepot and the level $l(t)$ of pot.

(d) What is $V_2'(t)$ at $l = 5$?

0.4 Linearization (or standard linear approximation) and Differential

The linearization $L(x) = f(a) + f'(a)(x - a)$ is originally the tangential line to f at $x = a$. So, we think that for x near a,

$$L(x) \approx f(x).$$

While $\Delta y = f(x) - f(a) \approx f'(a)\Delta x = f'(a)(x - a)$,

$$\Delta y \approx dy = L(x) - f(x) = f'(a)dx$$

where

$$dx = \Delta x = (x - a).$$

Thus, dy is an estimate for Δy by the linearization.
13.

(a) Find the linearization of $y = f(x) = \sqrt{1 + x}$ at $x = 0$.

(b) Find dy.

(c) Find dy when $x = 0$ and $dx = 0.2$.

(d) Estimate $f(0.2)$ by the linearization.