Homework 11 (due on 11/15)

- Midterm 2 is Monday, November 18 in lecture. It will cover Sections 14-15, 17-20, 24-25, and 28. A sample exam will be posted.
- Read Section 29 for the next week.

25.3 Let $f_n(x) = \frac{n+\cos x}{2n+\sin^2 x}$ for all real numbers x.

(a) Show f_n converges uniformly on $\mathbb{R}$. Hint: First decide what the limit function is; then show (f_n) converges uniformly to it. Use $|\cos x|, |\sin x| \leq 1$.

25.6 (a) Show that if $\sum |a_n| < \infty$, then $\sum a_k x^k$ converges uniformly on $[-1, 1]$ to a continuous function.

25.7 Show $\sum_{n=1}^{\infty} \frac{1}{n^2} \cos(nx)$ converges uniformly on $\mathbb{R}$.

25.10 (a) Show $\sum \frac{x^n}{1+x^n}$ converges for $x \in [0, 1)$. Hint: Compare it with $\sum x^n$.

(b) Show that the series converges uniformly on $[0, a]$ for each a, $0 < a < 1$.

(c) Does the series converge uniformly on $[0, 1)$? Explain. Hint: We proved in class that $\sum x^n$ does not converge uniformly on $[0, 1)$.

28.4 Let $f(x) = x^2 \sin \frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$.

(a) Use Theorems 28.3 and 28.4 to show f is differentiable at each $a \neq 0$ and calculate $f'(a)$. Use, without proof, the fact that $\sin x$ is differentiable and that $\cos x$ is its derivative.

(b) Use the definition to show f is differentiable at $x = 0$ and $f'(0) = 0$.

(c) Show f' is not continuous at $x = 0$.

28.7 Let $f(x) = x^2$ for $x \geq 0$ and $f(x) = 0$ for $x < 0$.

(b) Show f is differentiable at $x = 0$ and calculate $f'(0)$. Hint: You will have to use the definition of derivative. You may first consider one-sided derivatives.

(c) Calculate $f'(x)$ for $x > 0$ and $x < 0$.

(d) Is f' continuous on $\mathbb{R}$? differentiable on $\mathbb{R}$? Explain.

28.9 Let $h(x) = (x^4 + 13x)^7$. (a) Calculate $h'(x)$.

28.10 Let $h(x) = (\cos x + e^x)^{12}$. (a) Calculate $h'(x)$. You may use the fact that $\cos x$ and e^x are differentiable and that $-\sin x$ and e^x are their derivatives.

28.11 Suppose f is differentiable at a, g is differentiable at $f(a)$, and h is differentiable at $g \circ f(a)$. State and prove the chain rule for $(h \circ g \circ f)'(a)$. Hint: Apply Theorem 28.4 twice.