Homework 7 (due on 10/18)

- Read Sections 17 and 18 for the next week.
- Wednesday, October 16, 2019, is the last day to drop courses with no grade reported.

14.2 (b) Determine the convergence of $\sum(-1)^n$ and justify your answer.

14.3 (f) Determine the convergence of $\sum \frac{100^n}{n!}$ and justify your answer.

14.6 (a) Prove that if $\sum |a_n|$ converges and (b_n) is a bounded sequence, then $\sum a_n b_n$ converges. Hint: Use Theorem 14.4.

14.12 Let (a_n) be a sequence such that $\liminf |a_n| = 0$. Prove that there is a subsequence (a_{n_k}) such that $\sum a_{n_k}$ converges. Hint: First show that for any $r > 0$, there are infinitely many n such that $|a_n| < r$. Use this to show that there is a subsequence (a_{n_k}) such that $|a_{n_k}| \leq \frac{1}{2^k}$ for any $k \in \mathbb{N}$. The index sequence (n_k) can be constructed by induction. Besides the inequality that we want, the (n_k) should also satisfy $n_1 < n_2 < n_3 < \cdots$.

E1 Prove that for any $r > 0$, $\frac{r^n}{n!} \to 0$ and $\frac{n!}{r^n} \to +\infty$. Hint: Use Ratio Test.

15.1 Determine which of the following series converge. Justify your answers.

$$(a) \sum \frac{(-1)^n}{n}; \qquad (b) \sum \frac{(-1)^n n!}{2^n}.$$