MTH 320 Section 004 Midterm 1 Sample

Instructions: You have 60 minutes to complete the exam. There are five problems, worth a total of 20 points. You may not use any books or notes. Partial credit will be given for progress toward correct proofs.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

NAME: ____

Note. This is a sample exam. The actual exam problems may not be similar to the sample exam problems, and may be harder. But they are sll similar to homework problems.

1. Prove the inequality:

$$||x| - |y|| \leq |x - y|, \quad \forall x, y \in \mathbb{R}.$$

2. (a) For a nonempty set $S \subseteq \mathbb{R}$, define $\min S$ and $\inf S$.

   (b) Let A and B be two nonempty subsets of $\mathbb{R}$. Prove that

$$\inf(A \cup B) = \min\{\inf A, \inf B\}.$$

   Here if $\inf A$ or $\inf B$ equals $-\infty$, then we understand $\min\{-\infty, -\infty\}$ and $\min\{-\infty, x\}$ for any $x \in \mathbb{R}$ as $-\infty$.

3. (a) When do we say that a sequence (s_n) converges to s?

   (b) Determine the limit of the sequence $s_n = \frac{\cos(n^2)}{2^n}$ and prove your claim.

4. (a) Give the definition of $\limsup s_n$ for a sequence (s_n).

   (b) Prove that for any two sequences of nonnegative numbers (s_n) and (t_n),

$$\limsup(s_n + t_n) \leq \limsup s_n + \limsup t_n.$$

   Here if $\limsup s_n$ or $\limsup t_n = +\infty$, we understand $(+\infty) + (+\infty)$ and $(+\infty) + x$ for any $x \in \mathbb{R}$ as $+\infty$.

5. Let (s_n) be a sequence defined recursively by $s_1 = 10$ and $s_n = \frac{1}{4}(s_{n-1} + 6)$.

(a) Show that (s_n) is decreasing and satisfies $s_n > 2$ for all n.

(b) Does (s_n) converge? If so, what is the limit? Justify your answer carefully.

This page is for scratch work. Feel free to tear it off. Do not write anything you want graded on this page unless you indicate very clearly that this is the case on the page of the corresponding problem.