Name: ______________________________

Section: ____________ **Recitation Instructor**: ______________________________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 13.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page. You must indicate if you desire work on the back of a page to be graded.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ______________________________

SIGNATURE

Standard Response Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

1. Determine if the following series converge or diverge. **If the series converges, also compute the sum**. You must show all of your work and support your conclusions.

(a) (7 points) $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n-1}}{2^{2n}}$

Solution:

Use the **ratio test**:

Since all terms are positive for $n \geq 1$, we don't need absolute value

$$\frac{a_{n+1}}{a_n} = \frac{3^n}{2^{2n+2}} \cdot \frac{2^{2n}}{3^{n-1}} = \frac{3}{4}$$

So $L = \lim_{n\to\infty} \dfrac{a_{n+1}}{a_n} = \dfrac{3}{4} < 1$ and the series $\boxed{\text{converges}}$.

Alternatively, $\displaystyle\sum_{n=0}^{\infty} a_n = \frac{1}{3}\sum_{n=0}^{\infty} \frac{3^n}{2^{2n}} = \frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n$, so it is a geometric series of ratio $\dfrac{3}{4} < 1$ and the series $\boxed{\text{converges}}$.

Since $\displaystyle\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$, in our case $a = 1$ and $r = \dfrac{3}{4}$, so $\displaystyle\frac{1}{3}\left\{\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n\right\} = \frac{1}{3}\left\{\frac{1}{1-\frac{3}{4}}\right\} = \boxed{\frac{4}{3}}$.

(b) (7 points) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2+5}}{3n+4}$

Solution:

Consider the **test for divergence**:

Let $a_n = \dfrac{\sqrt{n^2+5}}{3n+4}$.

$$\begin{aligned}\lim_{n\to\infty} a_n &= \lim_{n\to\infty} \frac{\sqrt{n^2+5}}{3n+4} \\ &= \lim_{n\to\infty} \frac{\sqrt{1+\frac{5}{n^2}}}{3+\frac{4}{n}} = \lim_{n\to\infty} \frac{\sqrt{1+0}}{3+0} = \frac{1}{3} \neq 0\end{aligned}$$

Therefore, by the test for divergence, the series $\displaystyle\sum_{n=1}^{\infty} a_n$ $\boxed{\text{diverges.}}$

2. Determine if the following series converge or diverge. You must show all of your work and justify your use of any series convergence tests.

(a) (7 points) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2+5}{2^n}$

Solution:

Use the **ratio test**:

Since all terms are positive for $n \geq 1$, we don't need absolute value

$$\begin{aligned}
\frac{a_{n+1}}{a_n} &= \frac{3(n+1)^2+5}{2^{n+1}} \cdot \frac{2^n}{3n^2+5} \\
&= \frac{3(n+1)^2+5}{3n^2+5} \cdot \frac{1}{2} \\
&= \left(\frac{3n^2+6n+8}{3n^2+5}\right) \cdot \frac{1}{2} \\
&= \left(\frac{3+\frac{6}{n}+\frac{8}{n^2}}{3+\frac{5}{n^2}}\right) \cdot \frac{1}{2}
\end{aligned}$$

So $L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1}{2}$ and the series $\boxed{\text{converges}}$.

(b) (7 points) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n-1}$

Solution:

Consider the **limit comparison test**:

All terms are positive for $n \geq 1$, so we may apply the test.

Let $a_n = \dfrac{1}{3^n-1}$ and $b_n = \dfrac{1}{3^n}$

$$\begin{aligned}
\lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \frac{1}{3^n-1} \cdot \frac{3^n}{1} \\
&= \lim_{n\to\infty} \frac{3^n}{3^n-1} \\
&= \lim_{n\to\infty} \frac{1}{1-\frac{1}{3^n}} = \frac{1}{1-0} = 1
\end{aligned}$$

Therefore, by the limit comparison test, either both a_n and b_n converge or they both diverge. But b_n is a geometric series with ratio $= \dfrac{1}{3}$ which converges. Therefore the series $\boxed{\text{converges.}}$

3. For each of the following functions, find its 3rd degree Taylor polynomial centered at the given a.

   (a) (7 points) $f(x) = \sin(5x)$, centered at $a = 0$.

Solution:

We know that $\sin(t) = t - \frac{t^3}{6} + HOT$ (Higher Order Terms, i.e. terms with powers of the variable t higher than 3) is the Taylor series at $t = 0$ (i.e. MacLaurin series) of $\sin(t)$.

So, substituting $t = 5x$, we get $\sin(5x) = 5x - \frac{5^3x^3}{6} + HOT$.

Since this is a power series centered at $x = 0$ which equals the function $\sin(5x)$ for all x in some open interval containing $x = 0$, it has to be the Taylor series of that function at $x = 0$. (Recall the derivation of the Taylor series formula.)

So the 3rd degree Taylor polynomial of the function $\sin(5x)$ is $\boxed{T_3(x) = 5x - \frac{5^3x^3}{6}.}$

   (b) (7 points) $g(x) = \ln(x)$, centered at $a = 2$.

Solution:

$$\begin{aligned}
g(x) = \ln(x) &\longrightarrow g(2) = \ln(2) \\
g'(x) = \frac{1}{x} &\longrightarrow g'(2) = \frac{1}{2} \\
g''(x) = \frac{-1}{x^2} &\longrightarrow g''(2) = \frac{-1}{4} \\
g'''(x) = \frac{2}{x^3} &\longrightarrow g'''(2) = \frac{1}{4}
\end{aligned}$$

So the 3rd degree Taylor polynomial of the function $g(x) = \ln(x)$ centered at $a = 2$ is

$$\begin{aligned}
T_3(x) &= g(2) + \frac{g'(2)}{1!}(x-2) + \frac{g''(2)}{2!}(x-2)^2 + \frac{g'''(2)}{3!}(x-2)^3 \\
&= \ln(2) + \frac{1}{2} \cdot \frac{1}{1!}(x-2) - \frac{1}{4} \cdot \frac{1}{2!}(x-2)^2 + \frac{1}{4} \cdot \frac{1}{3!}(x-2)^3 \\
&= \ln(2) + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \frac{1}{24}(x-2)^3.
\end{aligned}$$

I.e. the solution is $\boxed{T_3(x) = \ln(2) + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \frac{1}{24}(x-2)^3.}$

4. (7 points) Find the Maclaurin series (Taylor series centered at $a = 0$) representation of $f(x) = \dfrac{3x^2}{1+x^2}$. Express your answer in sigma notation.

Solution:
$\dfrac{3x^2}{1+x^2} = \dfrac{a}{1-r}$ with $a = 3x^2$ and $r = -x^2$.

So $f(x)$ can be written as the geometric series. $\boxed{\sum_{n=0}^{\infty} ar^n = \sum_{n=0}^{\infty} 3x^2(-x^2)^n = \sum_{n=0}^{\infty} (-1)^n 3x^{2n+2}.}$

5. (7 points) Find the interval of convergence for the power series $\displaystyle\sum_{n=3}^{\infty} \frac{(7x-2)^n}{(n+3)^2}$. (Leave your answer as an open interval; you do not have to test the end points for convergence.)

Solution: Since $\displaystyle\sum_{n=3}^{\infty} \frac{(7x-2)^n}{(n+3)^2} = \sum_{n=3}^{\infty} \frac{7^n}{(n+3)^2} \cdot \left(x - \frac{2}{7}\right)^n$, the center of the interval of convergence is the center of the power series, i.e. $x = \dfrac{2}{7}$.

To find the radius of convergence, we apply the ratio test with $a_n = \dfrac{7^n}{(n+3)^2} \cdot \left(x - \dfrac{2}{7}\right)^n$.

$$\begin{aligned}
\left|\frac{a_{n+1}}{a_n}\right| &= \frac{7^{n+1}}{(n+4)^2} \cdot \left|\left(x - \frac{2}{7}\right)^{n+1}\right| \cdot \frac{(n+3)^2}{7^n} \cdot \left|\left(x - \frac{2}{7}\right)^n\right| \\
&= \left(\frac{n+3}{n+4}\right)^2 \cdot 7 \cdot \left|x - \frac{2}{7}\right| \\
&= \left(\frac{1+\frac{3}{n}}{1+\frac{4}{n}}\right)^2 \cdot 7 \cdot \left|x - \frac{2}{7}\right|
\end{aligned}$$

So $L = \lim_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = 7 \cdot \left|x - \dfrac{2}{7}\right|$. For the series to converge by the ratio test, we want $L < 1$, i.e. $7 \cdot \left|x - \dfrac{2}{7}\right| < 1$, which holds if and only if $\left|x - \dfrac{2}{7}\right| < \dfrac{1}{7}$, i.e. if x is in $\left(\dfrac{2}{7} - \dfrac{1}{7}, \dfrac{2}{7} + \dfrac{1}{7}\right)$.

In other words, the interval of convergence (ignoring the convergence at the endpoints) is $\boxed{\left(\dfrac{1}{7}, \dfrac{3}{7}\right)}$.

Multiple Choice. Circle the best answer. No work needed. No partial credit available.

6. (4 points) Find the limit of the **sequence** a_n where the n^{th} term is given by $a_n = \dfrac{3n + \cos(3n)}{4n}$.

   A. 0

   B. $\frac{3}{4}$

   C. 3

   D. 4

   E. The sequence does not have a limit.

7. (4 points) Which statement about the series $\sum_{n=2}^{\infty} \dfrac{\ln(3n)}{\sqrt{n^2 - 1}}$ is true?

   A. It **diverges** by using a comparison test with $\sum_{n=1}^{\infty} \dfrac{1}{n}$.

   B. It **converges** by using a comparison test with $\sum_{n=1}^{\infty} \dfrac{1}{n}$.

   C. It **diverges** by using a comparison test with $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$.

   D. It **converges** by using a comparison test with $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$.

   E. It **diverges** by the ratio test.

8. (4 points) Which statement is true about the series $\sum_{n=2}^{\infty} \dfrac{1}{n(\ln(n))^p}$?

   A. The **integral test** shows that the series converges for all p.

   B. The **integral test** shows that the series diverges for $p \leq 1$.

   C. The **integral test** hypotheses are not met by this series, so it cannot be applied.

   D. The **integral test** hypotheses are met by this series, however the test is inconclusive.

   E. None of the above are true.

9. (4 points) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{(2x+3)^{4n+5}}{(n-1)!}$.

   A. 1/4

   B. 1/3

   C. 3

   D. 4

   E. $+\infty$

10. (4 points) Determine whether the following series are absolutely convergent, conditionally convergent, or divergent:

$$(1)\ \sum_{n=3}^{\infty} \frac{1}{n^2 \ln(n)} \quad \text{and} \quad (2)\ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$

   A. (1) is absolutely convergent; (2) is divergent.

   B. (1) is conditionally convergent; (2) is divergent.

   C. (1) is absolutely convergent; (2) is conditionally convergent.

   D. (1) is divergent; (2) is conditionally convergent.

   E. Both (1) and (2) are conditionally convergent.

11. (4 points) Which statement is true about the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$?

   A. By the ratio test, the series converges.

   B. By the ratio test, the series diverges.

   C. The ratio test is inconclusive for this series, but the series converges by another test.

   D. The ratio test is inconclusive for this series, but the series diverges by another test.

   E. None of the above are true.

12. (4 points) By Taylor's Inequality, on the interval $[-2, 2]$, the difference between e^x and its degree 2 Taylor polynomial centered at $x = 0$ is at most:

A. $\frac{|x|^2}{2}$.

B. $\frac{|x|^3}{6}$.

C. $\frac{e^2|x|^3}{6}$.

D. $\frac{|x|^4}{24}$.

E. $\frac{e^3|x|^4}{24}$.

13. (4 points) The Taylor series of the function $f(x)$, centered at $a = 2$, is given by $\sum_{n=0}^{\infty} \frac{n^2+5}{n!}(x-2)^n$. What is the value of the third derivative $f'''(2)$?

A. 5

B. 9/2

C. 9

D. 14/6

E. 14

14. (4 points) The Taylor series up to order 4, centered at $a = 0$, for $f(x) = \ln(1+x) - x\sin(x)$ is

A. $1 + x - \frac{3x^2}{2} + \frac{x^3}{2} - \frac{x^4}{4}$

B. $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$

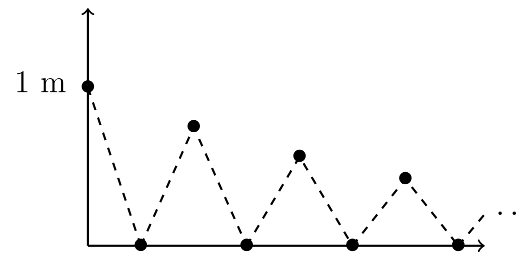
C. $x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{4}$

D. $x - \frac{3x^2}{2} + \frac{x^3}{3} - \frac{x^4}{12}$

E. $x - \frac{3x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4!}$

More Challenging Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

15. (a) (4 points) A ball falls from a height of 1m and continues bouncing forever. Each time it hits the floor, it bounces back to $\frac{3}{4}$ of the previous height. Write a (numerical) series that represents the total *vertical* distance traveled by the ball (ignore any horizontal displacement). (Hint: don't forget to include the distance traveled before the first bounce.)



Solution:
The vertical distance travelled before the first bounce is 1. Then the first bounce reaches a height of $\frac{3}{4}$, the second bounce reaches a height of $\frac{3}{4}$ times the previous bounce, i.e. $\left(\frac{3}{4}\right)^2$, the third bounce reaches a height of $\left(\frac{3}{4}\right)^3$, the fourth bounce reaches a height of $\left(\frac{3}{4}\right)^4$, and so on, so that the n-th bounce reaches a height of $\left(\frac{3}{4}\right)^n$. For each bounce, the ball goes up and down, i.e. it travels twice the height of the bounce.

So the $\boxed{\text{total vertical distance} = 1 + 2\cdot\left(\frac{3}{4}\right) + 2\cdot\left(\frac{3}{4}\right)^2 + 2\cdot\left(\frac{3}{4}\right)^3 + \cdots = 1 + 2\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n.}$

(b) (4 points) Compute the sum of the series you found in part (a).

Solution: $\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n = \sum_{n=0}^{\infty}\frac{3}{4}\cdot\left(\frac{3}{4}\right)^n = \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ with $a = \frac{3}{4}$ and $r = \frac{3}{4}$, so

$$1 + 2\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n = 1 + 2\cdot\frac{\frac{3}{4}}{1-\frac{3}{4}} = \boxed{7}.$$

16. (6 points) By the ratio test, the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$ is found to be $R = 2$.

If $b_n = (a_n)^2$, does the power series $\sum_{n=1}^{\infty} b_n x^n$ converge at $x = 3$? Justify your reasoning.

Solution: The ratio test requires checking $L = \lim_{n\to\infty}\left|\frac{a_{n+1}x^{n+1}}{a_n x^n}\right| = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right||x|$. If the radius of convergence is $R = 2$ and it has been found by the ratio test, then this means that $L < 1$ precisely for

$|x| < 2$, i.e. that

$\widetilde{L} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2}$.

Then $\widetilde{M} = \lim_{n\to\infty} \left|\frac{b_{n+1}}{b_n}\right| = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|^2 = \frac{1}{4}$, so, again by the ratio test, the series $\sum_{n=1}^{\infty} b_n x^n$ converges whenever $M = \lim_{n\to\infty} \left|\frac{b_{n+1}x^{n+1}}{b_n x^n}\right| = \lim_{n\to\infty} \left|\frac{b_{n+1}}{b_n}\right| |x| < 1$, i.e. it converges at least in the interval $(-4, 4)$.

(Note: The wording "at least" is because we are not checking the endpoints of the interval $(-4, 4)$, where the series might converge as well.)

Thus $\boxed{\text{the series } \sum_{n=1}^{\infty} b_n x^n \text{ converges at } x = 3.}$

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	12	
7	12	
8	12	
9	14	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET PAGE 1

Integrals

- **Volume**: Suppose $A(x)$ is the cross-sectional area of the solid S perpendicular to the x-axis, then the volume of S is given by

$$V = \int_a^b A(x)\ dx$$

- **Work**: Suppose $f(x)$ is a force function. The work in moving an object from a to b is given by:

$$W = \int_a^b f(x)\ dx$$

- $\int \frac{1}{x}\ dx = \ln|x| + C$
- $\int \tan x\ dx = \ln|\sec x| + C$
- $\int \sec x\ dx = \ln|\sec x + \tan x| + C$
- $\int a^x\ dx = \frac{a^x}{\ln a} + C \qquad$ for $a \neq 1$
- **Integration by Parts**:

$$\int u\ dv = uv - \int v\ du$$

- Arc Length Formula:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\ dx$$

Derivatives

- $\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$
- Inverse Trigonometric Functions:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

- If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Hyperbolic and Trig Identities

- Hyperbolic Functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch}(x) = \frac{1}{\sinh x}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech}(x) = \frac{1}{\cosh x}$$

$$\tanh(x) = \frac{\sinh x}{\cosh x} \qquad \coth(x) = \frac{\cosh x}{\sinh x}$$

- $\cosh^2 x - \sinh^2 x = 1$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2\sin x \cos x$
- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

FORMULA SHEET PAGE 2

Series

- **nth term test for divergence:** If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.
- **The p-series:** $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$.
- **Geometric:** If $|r| < 1$ then $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$
- **The Integral Test:** Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then
  - (i) If $\int_1^{\infty} f(x)\, dx$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent.
  - (ii) If $\int_1^{\infty} f(x)\, dx$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.
- **The Comparison Test:** Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.
  - (i) If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent.
  - (ii) If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum a_n$ is also divergent.
- **The Limit Comparison Test:** Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

where c is a finite number and $c > 0$, then either both series converge or both diverge.

- **Alternating Series Test:** If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ satisfies
  - (i) $0 < b_{n+1} \leq b_n$ for all n
  - (ii) $\lim_{n\to\infty} b_n = 0$

  then the series is convergent.
- **The Ratio Test**
  - (i) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.
  - (ii) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.
  - (iii) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$, the Ratio Test is inconclusive.
- **Maclaurin Series**: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$
- **Taylor's Inequality** If $|f^{(n+1)}(x)| \leq M$ for $|x-a| \leq d$, then the remainder $R_n(x)$ of the Taylor series satisfies the inequality

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1} \quad \text{for } |x-a| \leq d$$

- Some Power Series
  - $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ $R = \infty$
  - $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ $R = \infty$
  - $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ $R = \infty$
  - $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ $R = 1$
  - $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ $R = 1$