Disclaimer: This Final Exam Study Guide is meant to help you start studying.
It is not necessarily a complete list of everything you need to know.

The MTH 133 final exam mainly consists of standard response questions where students must justify their work. In addition to these, the Final Exam may consist of: fill in the blank, true/false, or multiple choice questions.

Most instructors agree that a good way to study for the final is to do lots of problems to help familiarize yourself with all of the concepts covered.

Sections containing similar concepts have been grouped in blue boxes. Most MTH 133 final exam writers agree that the items below contain crucial material for showcasing MTH 133 knowledge and are therefore **very important**. Expect at least one problem from each group on the final exam.

Important Items from Each Section:

5.2 - Volumes

- Recall the formula for volume $V = \int_a^b A(x)\, dx$. Where $A(x)$ is the cross-sectional area perpendicular to x-axis.

5.4 - Work

- If $f(x)$ is a variable force function then the work done in moving the object from a to b is given by:

$$W = \int_a^b f(x)\, dx$$

- Recall **Hooke's Law** for springs $f(x) = kx$. Where x is the number of units beyond the spring's natural length.

Good Final Exam Review Problems:

- 5.2.7
- 5.2.9
- 5.2.39
- 5.3.9
- 5.3.15
- 5.3.42
- 5.4.8
- 5.4.13
- 5.4.21

Important Items from Each Section:

6.1 - Inverse Functions

- Recall the formula: $(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$.

6.2 - The Natural Logarithmic Function

- Recall the definition and properties of $\ln x$.
- Know that $\frac{d}{dx}(\ln|x|) = \frac{1}{x}$.
- Remember the steps in logarithmic differentiation.

6.3 - The Natural Exponential Function

- Recall the properties of e^x.

6.4 - General Logarithmic and Exponential Functions

- Know properties such as $a^x = e^{x \ln a}$ and $\log_a x = \frac{\ln x}{\ln a}$ to help you derive general logarithms and exponentials.
- Recall the limit: $\lim_{x \to 0}(1+x)^{1/x} = e = \lim_{n \to \infty}\left(1 + \frac{1}{n}\right)^n$

6.5 - Exponential Growth and Decay

- Know that the only solutions to the differential equation $dy/dt = ky$ are:

$$y(t) = y(0)e^{kt}$$

- Know the formulas for Radioactive Decay/Growth, Newton's Law of Cooling, and Compound Interest.

Good Final Exam Review Problems:

- 6.1.40
- 6.1.42
- 6.2.20
- 6.2.63
- 6.2.74
- 6.3.54
- 6.4.45
- 6.4.54
- 6.5.3
- 6.5.9
- 6.5.14

Important Items from Each Section:

6.6 - Inverse Trigonometric Functions

- Know the algebraic properties of trigonometric inverses.
- Using the Implicit Function Theorem, know how to derive the formulas:

$$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}},\quad \frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}},\quad \frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$$

- The derivative formulas above yield the antiderivative formulas:

$$\int \frac{1}{\sqrt{1-x^2}}\,dx=\sin^{-1}x+C,\quad \int \frac{1}{x^2+a^2}\,dx=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$$

6.7 - Hyperbolic Functions

- Know the definitions, identities, and derivatives of the hyperbolic functions.

Good Final Exam Review Problems:

- 6.6.23
- 6.6.30
- 6.6.49
- 6.7.38
- 6.7.39

Important Items from Each Section:

6.8 - Indeterminate Forms and L'Hospital's Rule

- Learn to recognize the following indeterminate forms when evaluating limits:
  - $0/0$
  - ∞/∞
  - $0\cdot\infty$
  - $\infty-\infty$
  - 0^0
  - ∞^0
  - 1^∞
- Understand how and when L'Hospital's Rule can be used to evaluate certain indeterminate limits.

Good Final Exam Review Problems:

- 6.8.17
- 6.8.22
- 6.8.44
- 6.8.53
- 6.8.56
- 6.8.61

Important Items from Each Section:

7.1 - Integration by Parts

- Recall the formula for integration by parts:

$$\int u\ dv = uv - \int v\ du$$

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Good Final Exam Review Problems:

- 7.1.5
- 7.1.15
- 7.1.18

Important Items from Each Section:

7.2 - Trigonometric Integrals

- Know the strategies for integrating $\int \sin^m x \cos^n x\ dx$.
- Know the strategies for integrating $\int \tan^m x \sec^n x\ dx$.

7.3 - Trigonometric Substitution

- Use the substitutions below to help evaluate integrals.

$$\sqrt{a^2 - x^2} \longleftrightarrow x = a\sin\theta, \qquad \sqrt{a^2 + x^2} \longleftrightarrow x = a\tan\theta, \qquad \sqrt{x^2 - a^2} \longleftrightarrow x = a\sec\theta$$

Good Final Exam Review Problems:

- 7.2.2
- 7.2.29
- 7.3.13
- 7.2.16
- 7.2.36
- 7.3.19
- 7.2.23
- 7.3.4
- 7.3.22

Important Items from Each Section:

7.4 - Integration of Rational Functions by Partial Fractions

- Recall how to perform long division of polynomials.
- Know the different cases for partial fractions and how/when to correctly apply each.

Good Final Exam Review Problems:

- 7.4.10
- 7.4.23
- 7.4.32
- 7.4.21
- 7.4.29
- 7.4.40

Important Items from Each Section:

7.8 - Improper Integrals

- Remember both types of improper integrals and know how to carefully evaluate each.
- Recall that $\int_1^\infty \frac{1}{x^p} dx$ is convergent if $p > 1$ and divergent if $p \leq 1$.
- Know the Comparison Test for and Limit Comparison Test for improper integrals.

Good Final Exam Review Problems:

- 7.8.10
- 7.8.13
- 7.8.31

Important Items from Each Section:

8.1 - Arc Length

- Know the formulas for arc length:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\ dx, \qquad L = \int_c^d \sqrt{1 + [g'(y)]^2}\ dy$$

Good Final Exam Review Problems:

- 8.1.9
- 8.1.12
- 8.1.20

Important Items from Each Section:

9.3 - Separable Equations

- Know that separable differential equations can be written as $h(y)\ dy = g(x)\ dx$.
- Be able to explicitly solve a variety of separable differential equations.

Good Final Exam Review Problems:

- 9.3.12
- 9.3.14
- 9.3.22

Important Items from Each Section:

10.1 - Curves Defined by Parametric Equations

- Be able to transform parametrized curves $x = f(t), y = g(t)$ to Cartesian equations.
- Recognize graphs or parametrized curves and the direction of movement.

10.2 - Calculus with Parametric Curves

- Know how to find tangent lines to parametric curves.
- Remember the arc length formula: $L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Good Final Exam Review Problems:

- 10.1.6
- 10.1.28
- 10.2.3
- 10.2.18
- 10.2.32
- 10.2.42

Important Items from Each Section:

10.3 - Polar Coordinates

- Know how to transform between Cartesian and Polar Coordinates.
- Know how to sketch polar graphs

10.4 - Areas and Lengths in Polar Coordinates

- Know the area formula for polar equations: $A = \int_a^b \frac{1}{2}[f(\theta)]^2\, d\theta$.
- Remember the arc length formula for polar equations: $L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$.

Good Final Exam Review Problems:

- 10.3.17
- 10.3.18
- 10.3.24
- 10.3.33
- 10.3.37
- 10.4.9
- 10.4.20
- 10.4.47
- 10.4.48

Important Items from Each Section:

11.1 - Sequences

- Know the limit properties of sequences (e.g., sum, product, etc.).
- Remember the theorem:
  The sequence $\{r^n\}$ is convergent if $-1 < r \leq 1$ and divergent for all other values of r.

$$\lim_{n\to\infty} = \begin{cases} 0 & \text{if } -1 < r < 1 \\ 1 & \text{if } r = 1 \end{cases}$$

- Know the definitions of increasing, decreasing, monotonic, and bounded.

11.2 - Series

- Recall that if $|r| < 1$ then the geometric series $\sum_{n=0}^{\infty} ar^n$ is convergent and

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$$

- Know the test for divergence theorem:

$$\text{If } \lim_{n\to\infty} a_n \neq 0, \text{ then the series } \sum_{n=1}^{\infty} a_n \text{ is divergent.}$$

- Recall sum and scalar multiple equations for summations.

11.3 - The Integral Test and Estimates of Sums

- Be able to use **The Integral Test** for series convergence.
- Know the **p-series Test** for convergence.

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \text{ is convergent if } p > 1 \text{ and divergent if } p \leq 1.$$

11.4 - The Comparison Tests

- Know **The Comparison Test** and **The Limit Comparison Test** for series convergence.

11.5 - Alternating Series

- Be familiar with **The Alternating Series Test** and use it to determine when alternating series converge.

11.6 - Absolute Convergence and the Ratio and Root Tests

- Recall the definitions of absolutely convergent and conditionally convergent.
- Remember that absolutely convergent series are convergent.
- Know **The Ratio Test** and **The Root Test** for series convergence.

Good Final Exam Review Problems:

- 11.1.25
- 11.1.47
- 11.7.1-37 odd

Important Items from Each Section:

11.8 - Power Series

- Recall the definitions for power series, radius of convergence, and interval of convergence.
- Know the strategy for finding a power series' interval of convergence (typically using the ratio test).

11.9 - Representations of Functions as Power Series

- Know the formula:
$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$$
- Know how to integrate or differentiate power series using **term-by-term differentiation and term-by-term integration**.

11.10 - Taylor and Maclaurin Series

- Know that if f has a power series representation $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ then the coefficients c_n are given by:
$$c_n = \frac{f^{(n)}(a)}{n!}$$
- Recall **Taylor's Inequality** for finding the maximum error in approximating a function with a Taylor Polynomial.
- Know the formulas from Table 1 in 11.10 such as: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, for all x.

11.11 - Applications of Taylor Polynomials

- Use Taylor's formula to estimate the accuracy of the approximation $f(x) \approx T_n(x)$ in a given interval.

Good Final Exam Review Problems:

- 11.8.15
- 11.8.23
- 11.8.29
- 11.9.7
- 11.9.13
- 11.9.26
- 11.10.8
- 11.10.12
- 11.10.14
- 11.10.17
- 11.10.20
- 11.10.64
- 11.11.14
- 11.11.18
- 11.11.19