Complex Analysis
Qualifying Exam

Student ID: _____

August 20, 2025

This exam contains 7 pages (including this cover page) and 5 problems.

No calculators, computers, mobile phones, books, and lecture notes are allowed on the exam.

Write clearly and legibly. Points will be taken off if the grader cannot read the answer or understand the logical sequence.

Answer each question on its own page. Use the back of pages and/or the extra sheet at the end if you need more space. Clearly indicate when you do this.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (10 points) Let f be analytic on $\mathbb{C}$ and there exist real $M > 0$, $R > 0$ and integer $n \geq 1$ such that $|f(z)| \leq M|z|^n$ for all $|z| > R$. Prove that f is a polynomial of degree at most n.

2. (10 points) Use Abel's theorem to prove the identity

$$\sum_{n=1}^{\infty} \frac{\sin(n\varphi)}{n} = \frac{\pi - \varphi}{2}$$

for any $0 < \varphi < 2\pi$. The series on the left-hand side converges and you do not need to prove it.

Abel's theorem: Let $\sum_{n=0}^{\infty} a_n z^n$ have radius of convergence 1 and let $\sum_{n=0}^{\infty} a_n$ converge. Then

$$\lim_{r \to 1-} \sum_{n=0}^{\infty} a_n r^n = \sum_{n=0}^{\infty} a_n,$$

where $r \to 1-$ means that r tends to 1 along the real interval $(0, 1)$.

3. (10 points) Compute the integral

$$\int_{-\infty}^{\infty} \frac{x \sin(2x)}{x^2 + 9} dx.$$

4. (10 points) Compute the following integral, where γ is the unit circle centered at the origin and with a counterclockwise orientation,

$$\int_\gamma \frac{z^3 - 2z^2 + z - 1}{z^{25}(z - 1/2)^{100}(z - 2)} dz.$$

5. (10 points) Find at least one value $a > 0$ such that there are exactly 6 roots of the polynomial $33z^7 - z^5 + az^3 - 4z + 1$ in the annulus $\{z \in \mathbb{C} : \frac{1}{2} < |z| < 2\}$.