Note: all statements require proofs. You can make references to standard theorems from the course; however, you need state the relevant part of the theorem in your own words, unless it is a well-known named theorem. For example, "we had a theorem in the class that said that any continuous function on a compact subset of $\mathbb{R}^n$ is uniformly continuous" is a good reference, and "by the uniqueness theorem from the class, f is unique" is not a good reference.

1. Let $z_1, \ldots, z_n \in \mathbb{C}$ be distinct points and suppose f is injective and holomorphic on $\mathbb{C} \setminus \{z_1, \ldots, z_n\}$. Show that $f(z) = \frac{az+b}{cz+d}$ for some $a, b, c, d \in \mathbb{C}$.

2. Compute the following integral
$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}\, dx.$$

3. Determine the number of zeros the polynomial $p(z) = z^7 + 16z^2 + 1$ has in the open annulus $\{z \in \mathbb{C} \colon \frac{1}{2} < |z| < 2\}$.

4. Suppose f is a conformal map (i.e. a bijective holomorphic function) from the punctured disc $\mathbb{D} \setminus \{0\}$ to itself. Show that $f(z) = e^{i\theta} z$ for some $\theta \in \mathbb{R}$.

5. Let f be an entire function such that for each $n \in \mathbb{N}$
$$\frac{1}{2} e^{2^{(1.5n)}} \leq \max_{|z|=2^n} |f(z)| \leq e^{2^{(1.5n)}}.$$
Show that f has infinitely many zeros.