Numerical Analysis II
Qualify Exam
Fall, 2019

Note:

- **Cell phone, laptop or any electronic device with Wi-Fi capability are prohibited.**
- **A simple calculator is allowed.**
- **One 8.5"x11" page note is allowed (2-sided page is fine).**
- **Please show all your work. An answer, correct or incorrect, without proper explanation will yield zero credit.**

1. (**20 points**) Consider solving the IVP: $y' = f(x, y), y(x_0) = y_0$.
   (a) If we view the Euler's method as approximation of $y(x)$ by the first two terms of its Taylor expansion, construct a numerical method using the first three terms of the Taylor expansion.
   (b) (b) Apply the method developed in (1) to : $y' = x^2 + y + 1$, $y(0) = 1$. If $y_0 = 1$, find y_1 and y_2, with step size $h = 0.1$.

2. **(20 points)** Consider the predictor-corrector method with the Euler's method as the predictor and the backward Euler's method as the corrector.
   (a) Is this method A-stable? Please explain your answer.
   (b) Is this method a 2-stage Runge-Kutta method of order 2? Please explain your answer.
   (c) Apply this method to : $y' = x + y,\ y(0) = 2$. If $y_0 = 2$, find y_1 and y_2, with step size $h = \frac{1}{2}$.

3. **(30 points)** Consider solving the IVP: $y' = f(x, y), y(x_0) = y_0$. Determine if the following numerical methods are convergent:
   (a) $y_{n+1} = y_n + \frac{h}{2}(f(x_n, y_n) + f(x_{n+1}, y_{n+1}))$;
   (b) $y_{n+1} = 3y_n - 2y_{n-1} + \frac{h}{2}(f(x_n, y_n) - 3f(x_{n-1}, y_{n-1}))$;
   (c) $y_{n+1} = \frac{-3}{2}y_n + 3y_{n-1} - \frac{1}{2}y_{n-2} + 3hf(x_n, y_n)$.

4. **(10 points)** Consider the following 2-stage Runge-Kutta method for solving first order ODE:
   $y' = f(x, y)$
   $y_{n+1} = y_n + h(\gamma_1 k_1 + \gamma_2 k_2)$
   $k_1 = f(x_n, y_n)$
   $k_2 = f(x_n + \alpha h,\ y_n + \beta h k_1)$

   Show that all the Second order 2-stage Runger-Kutta methods share the same absolute stability region.