THE TENTH ANNUAL HERZOG PRIZE EXAMINATION

November 6, 1982

Problem 1: (CWRU 1979 MC) For which positive integers N can you completely pack a $10 \times 10 \times 20$ rectangular box with $1 \times 1 \times N$ bricks?

Problem 2: (CWRU 1979 MC) Starting with the point $(1,1)$, construct a sequence of points P_n as follows: Draw the arc of the circle with center at the origin from P_k down to the positive x-axis. P_{k+1} is the midpoint of the chord of this arc. Find $\lim_{n\to\infty} \{P_n\}$.

Problem 3: (Fritz Herzog) Let ABC be a spherical triangle of area π on the unit sphere. Can the sphere be completely covered using four congruent copies of ABC?

Problem 4: (L.M. Kelly) Show that for some side E of a given closed planar polygon and some vertex V the foot of the perpendicular from V to the line containing E falls on E.

Problem 5: Find all positive continuous functions $f(x)$ such that

$$\int_0^1 \frac{dx}{f(x)} = 1 \Big/ \int_0^1 f(x)dx \ .$$

Problem 6: (AMM E. 2928) Compute $\lim_{x\to-\infty} \sum_0^\infty x^n/n^n$ (if it exists).