The traditional **Herzog competition** of year 2023 took place on November 4. Twenty MSU undergraduate students had three hours to solve six problems. The winners are

First prize:	Erik Brodsky
Second prize:	Minh Do Quang
	Zhan (Jenny) Zhan
Third Prize:	Aamierul Harith Helmi
	Hoang Phuc Nguen
Honorable:	William Carvalho
	Andrew Harms
	Thinh Quang Ly

Here is the list of problems:

Question 1. Find

$$\sum_{n=1}^{\infty} \frac{1}{(n+2)\sqrt{n} + n\sqrt{n+2}}$$

Question 2. Given that $\sum \frac{1}{a_n} = \infty$ and $\sum \frac{1}{b_n} = \infty$, where (a_n) and (b_n) are nondecreasing sequences of positive real numbers, is it true that $\sum \frac{1}{a_n + b_n} = \infty$?

Question 3. Consider the power series expansion (defined for sufficiently small $|x|$)

$$\frac{1}{1-2x-x^2} = \sum_{n=0}^{\infty} a_n x^n.$$

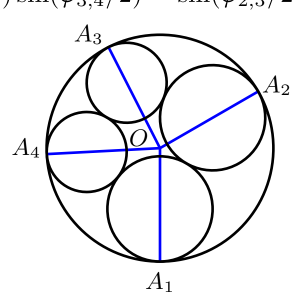
Prove that for any integer $n \geq 0$ there exists an integer m such that $a_n^2 + a_{n+1}^2 = a_m$.

Question 4. Define the function $C(n)$ on the set of natural numbers recurrently:
$C(1) = 1$; $C(2n) = (-1)^n C(n)$, $C(2n+1) = C(n)$. Find

$$\sum_{n=1}^{2023} C(n)C(n+2).$$

Question 5. Let us have four circles C_i, each of which is tangent to the (outer) unit circle centered at the point O and to its two neighbor circles as shown in the figure. Let A_i, $i = 1, 2, 3, 4$ be the points of intersection of the circles C_i with the unit circle. Denote by $\varphi_{i,j}$ angles A_iOA_j. Prove that

$$\sin(\varphi_{1,2}/2)\sin(\varphi_{3,4}/2) = \sin(\varphi_{2,3}/2)\sin(\varphi_{4,1}/2)$$



Question 6. Let a, b, and c be integer numbers. Prove that there exists a natural number n such that $\sqrt{n^3 + an^2 + bn + c}$ is not an integer.