THE FIRST HERZOG PRIZE EXAMINATION
10 November, 1973

Problem 1:

Let θ be the smallest vertex angle of a convex equilateral pentagon. What is the least possible value for θ?

Problem 2:

Compute the value of the integral

$$\int_0^\infty \frac{1}{1+x^\alpha} \cdot \frac{1}{1+x^2}\, dx$$

Problem 3:

n married couples gather for a picnic. As they arrive, some of the picnickers shake hands with one another, others do not. Of course, husband and wife do not shake hands. One of the picnickers, Mr. James, asks all the other persons (including Mrs. James) to tell him with how many people each of them shook hands. No two of the answers are the same. With how many people did Mrs. James shake hands?

Problem 4:

Show that the determinant of

$$\begin{pmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 1 & \cdots & 1 \\ 1 & 1 & 0 & 1 \cdots & 1 \\ & & & & \vdots \\ 1 & 1 & \cdots & 1 & 0 \end{pmatrix}$$

is non-zero.

Problem 5:

Compute the value of

a) $5^{1/5} \cdot 25^{1/25} \cdot 125^{1/125} \cdot \ldots$

b) $1 + \frac{\cos\theta}{1!} + \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} + \ldots$

Problem 6:

Let $P(x)$ be a polynomial with real coefficients with all roots real. If $P(x) \geq 0$ for all real x, then $P(x)$ is the square of a polynomial $Q(x)$ with real coefficients.