THE ELEVENTH ANNUAL HERZOG PRIZE EXAMINATION

November 5, 1983

Problem 1: (Fritz Herzog) Show that the numbers

$$4^n + n^4, \quad n = 2,3,4,5,\ldots$$

are all composite.

Problem 2: (John Kinney) Suppose we are given a loop of string of length sufficient to enclose S, a point set in the plane. A point of a pencil now draws a generalized ellipse keeping the string taut all the while. Will the tangent make equal angles with the two string segments at each point?

Problem 3: (L. Kelly) If the opposite edges of a tetrahedron are of equal length, then all the face angles are acute.

Problem 4: (T. Yen) Suppose we are given $2n+3$ points in the plane, no three collinear, and no four on a circle. Then there is a circle through three of these points containing exactly half the remaining points.

Problem 5: Do there exist differentiable functions h with

$$\cos h(x) = h(\sin x)$$

for all x?

(continued on back)

<u>Problem 6</u>: (C.R. MacCluer) Consider the ellipse

$$r = \frac{pe}{1 + e\cos\theta} .$$

Compute

$$\int_0^{2\pi} r \, d\theta, \int_0^{2\pi} r^2 \, d\theta, \text{ and } \int_0^{2\pi} r^3 \, d\theta ,$$

though not necessarily in that order.