THE TWENTY-FIRST HERZOG PRIZE EXAMINATION

November 13, 1993

1. Compute the length of an edge of a regular pentagon inscribed in a circle of unit radius. (Solve in radicals.)

2. Are there any angles θ for which $\sqrt{\sin\theta}$ and $\sqrt{\cos\theta}$ are both non-zero rational numbers?

3. Find all positive integers m and n for which

$$1! + 2! + \cdots + n! = m^2.$$

4. In a given tetrahedron the sum of the angles at each of its vertices is $180°$. Prove that all the faces of the tetrahedron are congruent.

5. Show that the equation

$$a^2x^n = x^{n-1} + x^{n-2} + \cdots + x + 1$$

has exactly one positive real solution.

6. Assume that $2 \leq x \leq y$. Prove or disprove that $y^{x+1} \leq xy^y$.