Name: _____ Signature: _____

- There are **no calculators** or notes allowed.
- You will be given exactly 90 minutes for this exam.
- No extra scratch paper is allowed. Use the back of the pages if needed.
- Please raise your hand if you have any questions and I will come to you.
- Good Luck!

Page	1	2	3	4	5	6	7	8	9	**Total**
Score										
Max	6	19	15	24	24	10	31	16	5	150

Fill in the Blank Questions. No partial credit available. No work needed.

1. (3 points) Let $f(x) = x^2 + 6x$. The slope of the secant line joining $P(1, f(1))$ and $Q(3, f(3))$ is _____.

2. (1+1+1=3 points) Let $f(x) = \begin{cases} x+3 & \text{if } x \leq -1 \\ 3 & \text{if } x > -1 \end{cases}$ Find following limits if they exist (if not, write DNE).

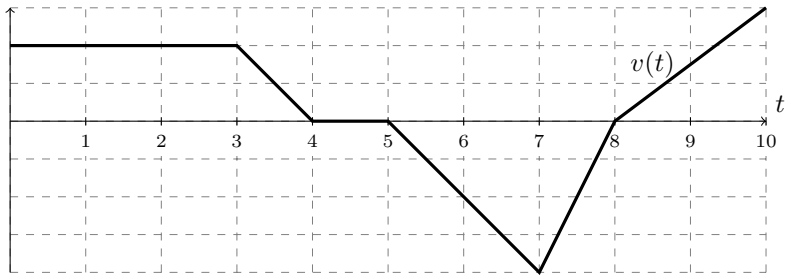
   (a) $\lim_{x \to -1^-} f(x) =$ _____

   (b) $\lim_{x \to -1^+} f(x) =$ _____

   (c) $\lim_{x \to -1} f(x) =$ _____

Extra room to work:

3. (3+3+3+3+3+4=19 points) The figure below shows the velocity $v(t)$ of a particle moving on a horizontal coordinate line, for t in a closed interval $[0, 10]$.



Fill in the following blanks. No partial credit available. No work needed. Use interval notation where appropriate.

(a) The particle is moving forward during: _____

(b) The particle's speed is increasing during: _____

(c) The particle has positive acceleration during: _____

(d) The particle has zero acceleration during: _____

(e) The particle achieves its greatest speed at: _____

(f) The particle stands still for more than an instant during: _____

Extra room to work:

Multiple choice questions. Circle the best answer available. No partial credit available.

4. (5 points) Find c such that the function $f(x) = \begin{cases} x^2 - 10, & x \leq c \\ 10x - 35, & x > c \end{cases}$ is continuous everywhere.

   (a) $c = 10$.

   (b) $c = 5$.

   (c) $c = \sqrt{10}$.

   (d) $c = 20$.

   (e) None of the above.

5. (5 points)$f(x) = \dfrac{x^2 - 9}{x^2 - 9x + 18}$ is continuous on:

   (a) $(-\infty, -3) \cup (-3, 3) \cup (3, 6) \cup (6, \infty)$.

   (b) $(-\infty, 6) \cup (6, \infty)$.

   (c) $(-\infty, 3) \cup (3, \infty)$.

   (d) $(-\infty, 3) \cup (3, 6) \cup (6, \infty)$.

   (e) None of the above.

6. (5 points) Consider proving that $\lim_{x \to 3}(2x - 5) = 1$ using the formal definition of the limit. The largest value of $\delta > 0$ so that the formal definition of the limit holds is:

   (a) $\delta = 2\epsilon$.

   (b) $\delta = 5\epsilon$.

   (c) $\delta = \dfrac{\epsilon}{2}$.

   (d) $\delta = \dfrac{\epsilon}{5}$.

   (e) None of the above.

Extra room to work:

Standard response questions. *SHOW ALL OF YOUR WORK.*

7. (8+8+8=24 points) Evaluate the limits

   (a) $\lim_{a \to 64} \dfrac{64 - a}{8 - \sqrt{a}}$

Answer:

   (b) $\lim_{x \to 2} \dfrac{x^2 + 7x - 18}{x^2 - 2x}$

Answer:

   (c) $\lim_{s \to 0} \dfrac{\sin(s^2 - 7s)}{s}$

Answer:

8. (8+8+8=24 points) Find the derivatives of the following functions. **You do not need to simplify**

(a) $f(y) = 6\tan(3\sin(y))$

Answer:

$f'(y) =$

(b) $g(x) = \left(\frac{1}{x} + 11\right)(4\sqrt{x} + 7)$

Answer:

$g'(x) =$

(c) $h(t) = \frac{-6\tan t + 2}{\sec t}$

Answer:

$h'(t) =$

9. (3+7=10 points) Consider $f(x) = x^4 + x - 1$.

   Use the Intermediate Value Theorem to show that $f(x)$ has at least one root.

   (a) The Intermediate Value Theorem can be applied because $f(x)$ is _____

   (b) Finish using the Intermediate Value Theorem to show $f(x)$ has a root.

10. (15 points) Use the limit definition of the derivative to find the velocity function $v(t) = s'(t)$ in the case where the distance function $s(t)$ is given by

$$s(t) = \sqrt{t+4}$$

Answer:

$s'(t) =$

11. (8+8=16 points) Suppose y and x satisfy the implicit equation $xy^3 + xy = 16$.

(a) Find $\dfrac{dy}{dx}$.

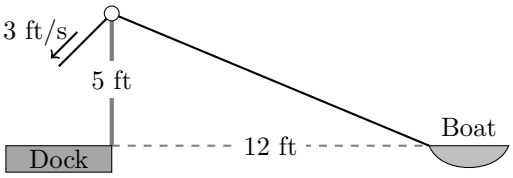
Answer:

$\dfrac{dy}{dx} =$

(b) Find the equation of the tangent line to the curve $xy^3 + xy = 16$ at the point $(8, 1)$.

Answer:

12. (16 points) A boat is pulled into a dock by a rope attached to the bow (front end) of the boat and passing through a pulley on the dock that is 5 ft higher than the bow of the boat. If the rope is pulled in at a rate of 3 ft/s, at what speed is the boat approaching the dock when it is 12 ft from the dock?



Answer:

13. (5 points) Use implicit differentiation to determine a derivative formula for $f(x) = \arccos(x)$

    Hint: $\arccos(x)$ *is defined to be the inverse of* $\cos(x)$

Answer:

$$\frac{d}{dx}(\arccos(x)) =$$