• There are **no calculators** or notes allowed.
• You will be given exactly 90 minutes for this exam.
• No extra scratch paper is allowed. Use the back of the pages if needed.
• Please raise your hand if you have any questions and I will come to you.
• Good Luck!

### Fill in the Blank Questions
No partial credit available. No work needed.

1. (3 points) Let \( f(x) = x^2 + 6x \). The slope of the secant line joining \( P(1, f(1)) \) and \( Q(3, f(3)) \) is \[ \text{_____} \].

2. (1+1+1=3 points) Let \( f(x) = \begin{cases} x+3 & \text{if } x \leq -1 \\ 3 & \text{if } x > -1 \end{cases} \)

Find following limits if they exist (if not, write DNE).

(a) \( \lim_{x \rightarrow -1^-} f(x) = \text{ _____ } \)

(b) \( \lim_{x \rightarrow -1^+} f(x) = \text{ _____ } \)

(c) \( \lim_{x \rightarrow -1} f(x) = \text{ _____ } \)

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**Extra room to work:**
3. (3+3+3+3+3+4=19 points) The figure below shows the velocity $v(t)$ of a particle moving on a horizontal coordinate line, for $t$ in a closed interval $[0, 10]$.

Fill in the following blanks. No partial credit available. No work needed. Use interval notation where appropriate.

(a) The particle is moving forward during: ________________________________

(b) The particle’s speed is increasing during: ________________________________

(c) The particle has positive acceleration during: ________________________________

(d) The particle has zero acceleration during: ________________________________

(e) The particle achieves its greatest speed at: ________________________________

(f) The particle stands still for more than an instant during: ________________________________

Extra room to work:
Multiple choice questions. Circle the best answer available. No partial credit available.

4. (5 points) Find $c$ such that the function $f(x) = \begin{cases} x^2 - 10, & x \leq c \\ 10x - 35, & x > c \end{cases}$ is continuous everywhere.

(a) $c = 10$.
(b) $c = 5$.
(c) $c = \sqrt{10}$.
(d) $c = 20$.
(e) None of the above.

5. (5 points) $f(x) = \frac{x^2 - 9}{x^2 - 9x + 18}$ is continuous on:

(a) $(-\infty, -3) \cup (-3, 3) \cup (3, 6) \cup (6, \infty)$.
(b) $(-\infty, 6) \cup (6, \infty)$.
(c) $(-\infty, 3) \cup (3, \infty)$.
(d) $(-\infty, 3) \cup (3, 6) \cup (6, \infty)$.
(e) None of the above.

6. (5 points) Consider proving that $\lim_{x \to 3} (2x - 5) = 1$ using the formal definition of the limit. The largest value of $\delta > 0$ so that the formal definition of the limit holds is:

(a) $\delta = 2\epsilon$.
(b) $\delta = 5\epsilon$.
(c) $\delta = \frac{\epsilon}{2}$.
(d) $\delta = \frac{\epsilon}{5}$.
(e) None of the above.

Extra room to work:
Standard response questions. *SHOW ALL OF YOUR WORK.*

7. (8+8+8=24 points) Evaluate the limits

(a) \( \lim_{a \to 64} \frac{64 - a}{8 - \sqrt{a}} \)

Answer:

(b) \( \lim_{x \to 2} \frac{x^2 + 7x - 18}{x^2 - 2x} \)

Answer:

(c) \( \lim_{s \to 0} \frac{\sin(s^2 - 7s)}{s} \)

Answer:
8. (8+8+8=24 points) Find the derivatives of the following functions. You do not need to simplify

(a) \( f(y) = 6 \tan(3 \sin(y)) \)

Answer:
\[
f'(y) =
\]

(b) \( g(x) = \left( \frac{1}{x} + 11 \right) (4\sqrt{x} + 7) \)

Answer:
\[
g'(x) =
\]

(c) \( h(t) = \frac{-6 \tan t + 2}{\sec t} \)

Answer:
\[
h'(t) =
\]
9. (3+7=10 points) Consider \( f(x) = x^4 + x - 1 \).

Use the Intermediate Value Theorem to show that \( f(x) \) has at least one root.

(a) The Intermediate Value Theorem can be applied because \( f(x) \) is ________________

(b) Finish using the Intermediate Value Theorem to show \( f(x) \) has a root.
10. (15 points) Use the limit definition of the derivative to find the velocity function \( v(t) = s'(t) \) in the case where the distance function \( s(t) \) is given by

\[
s(t) = \sqrt{t + 4}
\]

Answer:

\[
s'(t) = \]

11. (8+8=16 points) Suppose \( y \) and \( x \) satisfy the implicit equation \( xy^3 + xy = 16 \).

(a) Find \( \frac{dy}{dx} \)

Answer:

\[
\frac{dy}{dx} =
\]

(b) Find the equation of the tangent line to the curve \( xy^3 + xy = 16 \) at the point \((8,1)\).

Answer:
12. (16 points) A boat is pulled into a dock by a rope attached to the bow (front end) of the boat and passing through a pulley on the dock that is 5 ft higher than the bow of the boat. If the rope is pulled in at a rate of 3 ft/s, at what speed is the boat approaching the dock when it is 12 ft from the dock?

Answer:
13. (5 points) Use implicit differentiation to determine a derivative formula for \( f(x) = \arccos(x) \)

*Hint:* \( \arccos(x) \) is defined to be the inverse of \( \cos(x) \)

Answer:

\[
\frac{d}{dx} (\arccos(x)) = 
\]