1. (4 points) Find the most general antiderivative of $f(x) = x + 2\sin x$.

Solution:

$$\int x + 2\sin x \; dx = \frac{x^2}{2} - 2\cos x + C$$

2. (4 points) Evaluate: $\displaystyle\int_1^9 \frac{x-1}{\sqrt{x}}\; dx$.

Solution:

$$\begin{aligned}
\int_1^9 \frac{x-1}{\sqrt{x}}\; dx &= \int_1^9 \sqrt{x} - \frac{1}{\sqrt{x}}\; dx \\
&= \int_1^9 x^{1/2} - x^{-1/2}\; dx \\
&= \left[\frac{2}{3}x^{3/2} - 2x^{1/2}\right]_1^9 \\
&= \left[\frac{2}{3}(27) - 2(3)\right] - \left[\frac{2}{3} - 2\right] = \boxed{14 - \frac{2}{3}}
\end{aligned}$$

3. (6 points) Solve the initial value problem: $\dfrac{dy}{dx} = 5x^4 - 2x^5, \qquad y(0) = 4$

Solution:

$$\begin{aligned}
y &= x^5 - \frac{x^6}{3} + C \\
4 &= 0 - 0 + C \\
y &= \boxed{x^5 - \frac{x^6}{3} + 4}
\end{aligned}$$

4. (8 points) Find the absolute maximum and absolute minimum values of

$$f(x) = 5 + 27x - x^3$$

on the interval $[0, 4]$.

Solution:

$$f'(x) = 27 - 3x^2 = 3(3-x)(3+x)$$

So $x = 3$ is a critical point in $[0, 4]$. Therefore by examining

$$\begin{aligned} f(0) &= 5 \\ f(3) &= 5 + 81 - 27 = 59 \\ f(4) &= 5 + 108 - 64 = 49 \end{aligned}$$

we find 5 is the absolute minimum and and 59 is the absolute maximum

5. (6 points) Let $F(x) = \displaystyle\int_{x^3}^{4} \frac{1}{t^2+2}\,dt$. Find $F'(x)$ ***without actually finding*** $\mathbf{F}(\mathbf{x})$.

Solution:

$$\begin{aligned} F(x) &= \int_{x^3}^{4} \frac{1}{t^2+2}\,dt \\ F(x) &= -\int_{4}^{x^3} \frac{1}{t^2+2}\,dt \\ F'(x) &= -\left[\frac{1}{(x^3)^2+2}\right] \cdot (3x^2) \\ F'(x) &= \boxed{\frac{-3x^2}{x^6+2}} \end{aligned}$$

6. Suppose: $f(x) = \dfrac{x^2}{\sqrt{x+1}}$, $f'(x) = \dfrac{x(3x+4)}{2(x+1)^{3/2}}$, $f''(x) = \dfrac{3x^2+8x+8}{4(x+1)^{5/2}}$

(a) (1 point) What is the domain for f?

Solution: $x > -1$ or $(-1, \infty)$

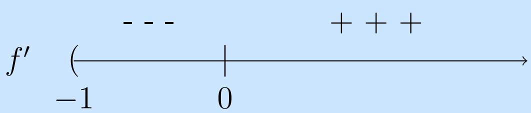
(b) (3 points) Write the equations for all vertical, horizontal, and slant asymptotes, if they exist.

Solution:

$$VA : x = -1$$
$$HA : NONE$$
$$SA : NONE$$

(c) (4 points) Identify the intervals over which $f(x)$ is increasing / decreasing.

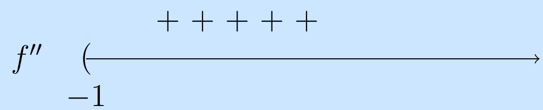
Solution: On its domain f' is never undefined and only 0 when $x = 0$. Using a number line and testing values we find



and so f is decreasing on $(-1, 0)$
and increasing on $(0, \infty)$.

(d) (4 points) Identify the intervals over which $f(x)$ is concave up / concave down.

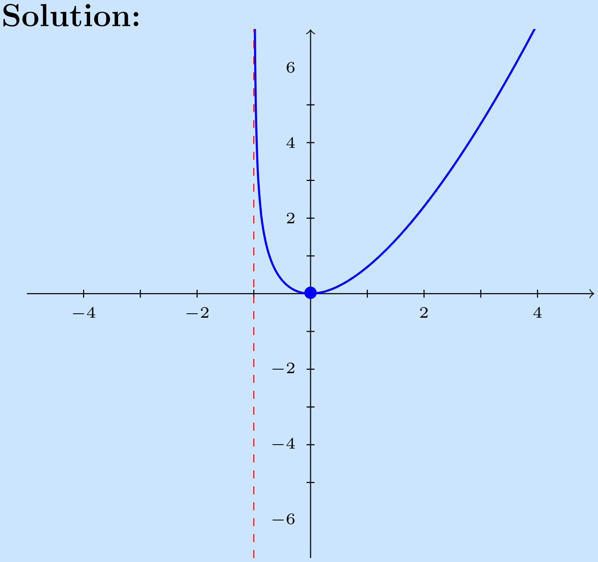
Solution: On its domain f'' is never undefined and never 0. Again, using a number line and testing values we find



and so f is never concave down
and concave up on $(-1, \infty)$.

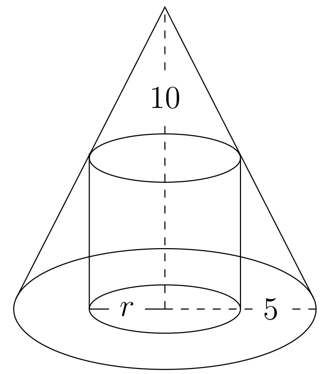
(e) (2 points) Sketch the curve of $y = f(x)$. *Parts (a)-(c) may be helpful.*

Solution:



7. A cylinder is inscribed in a right circular cone with a height of 10cm and a radius (at the base) of 5cm.

(a) (6 points) Express the volume of the cylinder V in terms of its radius r only.



Solution: Use the volume equation for a cylinder

$$V = h(\pi r^2)$$

And the fact that the cylinder is inscribed in the cone results in $h = 10 - 2r$ so therefore

$$V = \boxed{\pi(10-2r)r^2 = \pi(10r^2 - 2r^3)} \qquad (\text{for } r \in [0,5])$$

(b) (8 points) Find the maximum volume of such a cylinder. *Include units!*

Use techniques of calculus to justify that your answer is a maximum.

Solution: First search for critical points

$$\begin{aligned} V' &= \pi(20r - 6r^2) \\ 0 &= \pi(20r - 6r^2) \\ 6r &= 20 \\ r &= 10/3 \end{aligned}$$

Now using the closed interval method we have

$$\begin{aligned} V(0) &= 0 \\ V(10/3) &= \pi(10(100/9) - 2(1000/27) \\ V(5) &= 0 \end{aligned}$$

And so $\boxed{V = \pi\left(\frac{1000}{9} - \frac{2000}{27}\right) \text{ cm}^3}$ is the maximum.

8. (4 points) Use a linear approximation to estimate $\sqrt{35}$.

A. $6 - \frac{1}{4}$

B. $6 - \frac{1}{8}$

C. $6 - \frac{1}{9}$

D. $6 - \frac{1}{12}$

E. $6 - \frac{1}{20}$

9. (4 points) If $f(x) = x^2 - 3x$, which of the following statements is true by the Mean Value Theorem?

A. There is a value c in the interval $(0, 4)$ such that $f(c) = 1$.

B. There is a value c in the interval $(0, 4)$ such that $f'(c) = 1$.

C. There is a value c in the interval $(0, 4)$ such that $f(c) = 4$.

D. There is a value c in the interval $(0, 4)$ such that $f'(c) = 4$.

E. The Mean Value Theorem cannot be applied.

10. (4 points) Calculate $\int_0^5 |x - 2| \, dx$. *Hint: Sketch a graph.*

A. $\frac{13}{2}$

B. $\frac{25}{2} - 10$

C. $\frac{5}{2}$

D. $\frac{3}{2}$

E. None of the above

11. (4 points) Compute $\lim_{x\to\infty}(\sqrt{4x^2+3x}-2x)$

A. 0

B. $\frac{3}{4}$

C. 2

D. $\sqrt{3}$

E. ∞

12. (4 points) What is the horizontal asymptote of $f(x) = \dfrac{(3x-1)(x-2)}{(x+1)(2x)}$?

A. The function does not have a horizontal asymptote.

B. $y = 3$

C. $x = 3$

D. $y = \frac{3}{2}$

E. $x = \frac{3}{2}$

13. (4 points) The graph of a function $f(x)$ is shown below. What is the value of $\int_0^3 f(x)\,dx$?

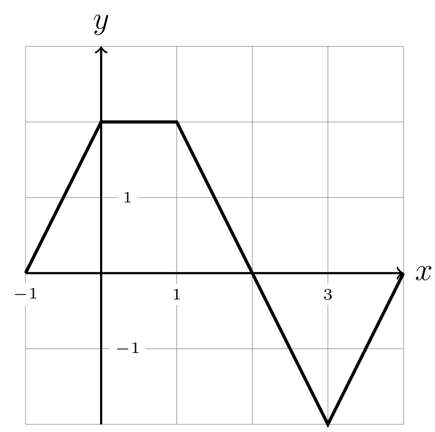
A. 0

B. 1

C. 2

D. 3

E. 4



14. (4 points) Evaluate the sum $\sum_{i=1}^{30}(3+2i)$

A. 1020

B. 930

C. 990

D. 555

E. 63

15. (4 points) Use Newton's Method to approximate a solution to the equation $x^5 + x = 35$ starting with $x_1 = 2$. Then $x_2 = ?$

A. $x_2 = -79$

B. $x_2 = 83$

C. $x_2 = 81$

D. $x_2 = 161/81$

E. $x_2 = 163/81$

16. (4 points) Find the slant asymptote of $y = \dfrac{x^2+5x+2}{x-1}$

A. $y = x - 1$

B. $y = x + 5$

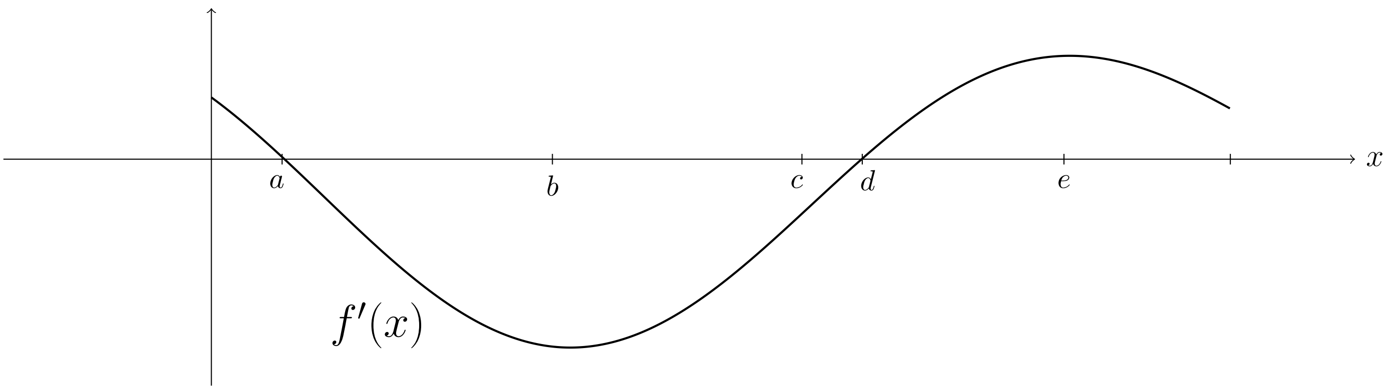
C. $y = x - 5$

D. $y = x + 6$

E. $y = x + 3$

More Challenging Question(s). Show all work to receive credit.

17. The graph of the **first derivative $\mathbf{f'(x)}$** is shown below.



(a) (4 points) List the critical points of $f(x)$ and determine if each is a local maximum, local minimum, or neither.

Solution: The critical points are $x = a$ and $x = d$. Because f' goes from positive to negative at $x = a$ we know this is a local maximum (by first derivative test) and because f' goes from negative to positive at $x = d$ it must be a local minimum.

(b) (4 points) List the inflection points of $f(x)$.

Solution: Inflection points of f are at $x = b$ and $x = e$ as this is where the f'' (the slope of f') changes signs.

(c) (2 points) *(Circle one)* **True** $f(b) > f(d)$.

(d) (4 points) Sketch a graph of $f(x)$ given that $f(0) = 1$.