Name: _____

Section: _____ **Recitation Instructor**: _____

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 11.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- You will be given exactly 90 minutes for this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: _____

SIGNATURE

FORMULA SHEET

Integrals

- **Volume**: Suppose $A(x)$ is the cross-sectional area of the solid S perpendicular to the x-axis, then the volume of S is given by

$$V = \int_a^b A(x)\ dx$$

- **Work**: Suppose $f(x)$ is a force function. The work in moving an object from a to b is given by:

$$W = \int_a^b f(x)\ dx$$

- $\displaystyle\int \frac{1}{x}\ dx = \ln|x| + C$
- $\displaystyle\int \tan x\ dx = \ln|\sec x| + C$
- $\displaystyle\int \sec x\ dx = \ln|\sec x + \tan x| + C$
- $\displaystyle\int a^x\ dx = \frac{a^x}{\ln a} + C \qquad \text{for } a \neq 1$
- **Integration by Parts**:

$$\int u\ dv = uv - \int v\ du$$

Derivatives

- $\dfrac{d}{dx}(\sinh x) = \cosh x \qquad \dfrac{d}{dx}(\cosh x) = \sinh x$
- Inverse Trigonometric Functions:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

- If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Hyperbolic and Trig Identities

- Hyperbolic Functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch}(x) = \frac{1}{\sinh x}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech}(x) = \frac{1}{\cosh x}$$

$$\tanh(x) = \frac{\sinh x}{\cosh x} \qquad \coth(x) = \frac{\cosh x}{\sinh x}$$

- $\cosh^2 x - \sinh^2 x = 1$
- $\cos^2 x + \sin^2 x = 1$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2\sin x \cos x$
- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

Standard Response Questions. Show all work to receive credit. Please **BOX** your final answer.

1. Evaluate the following limits. (If you use L'Hopital's Rule, explicitly state your reasoning.)

   (a) (9 points) $\lim_{t\to 0} \dfrac{\sin 4t}{e^{2t}-1}$

   (b) (9 points) $\lim_{x\to -\infty} xe^{x}$

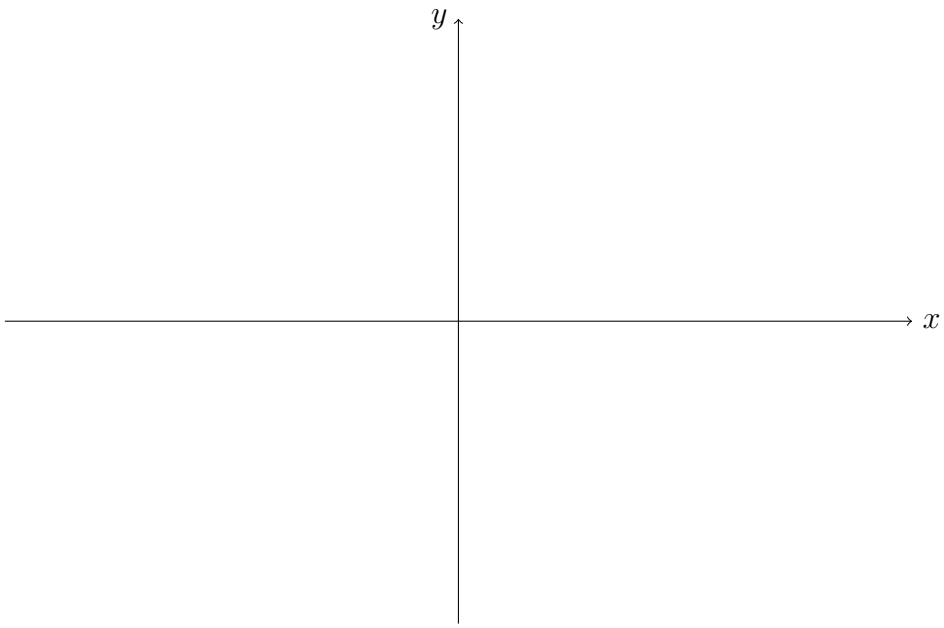
2. Evaluate the following integrals.

(a) (9 points) $\int \sin^3 x \ \cos^{3/2} x \ dx$

(b) (9 points) $\int \frac{5}{\sqrt{1-4x^2}} \ dx$

3. Find the volume of a solid obtained by rotating the region bounded by $y = 2\sin x$ and the x-axis from $x = 0$ to $x = \pi$ about the line $y = -2$.

   (a) (6 points) Draw a picture of the solid.

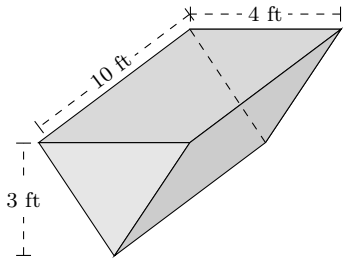


   (b) (6 points) Set up an integral the describes the volume of the solid.

   (c) (6 points) Evaluate the integral to find the volume of the solid.

4. (18 points) Evaluate $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$. Write your answer without using trigonometric functions.

5. (18 points) A tank (shown below) with ends that are isosceles triangles is filled with oil weighing 90 lbs/ft^3. Find the work required to pump all of the oil out to a height of 2 feet above the top of the tank. Express your answer in ft-lbs.



Multiple Choice. Circle the best answer. No work needed. No partial credit available.

6. (7 points) Evaluate $\displaystyle\int \frac{\cosh\sqrt{x}}{\sqrt{x}}dx$.

   A. $\frac{1}{2}\sinh\sqrt{x}+C$

   B. $-\frac{1}{2}\sinh\sqrt{x}+C$

   C. $2\sinh\sqrt{x}+C$

   D. $-2\sinh\sqrt{x}+C$

   E. None of the above.

7. (7 points) Let $f(x)=2x^3-4x^2+4$ for $x<0$. Find $\left(f^{-1}\right)'(-2)$ given $f(-1)=-2$.

   A. $1/14$

   B. 14

   C. $-1/14$

   D. $1/40$

   E. undefined

8. (7 points) Evaluate $\displaystyle\int xe^{2x}\,dx$.

   A. $x^2e^{2x}+C$

   B. $\dfrac{x}{2}e^{2x}-\dfrac{1}{4}e^{2x}+C$

   C. $xe^{2x}-e^{2x}+C$

   D. $\dfrac{1}{2}x^2e^{2x}+\dfrac{1}{2}xe^{2x}+C$

   E. $\dfrac{x^2}{2}e^{2x}-\dfrac{x^3}{6}e^{2x}+C$

9. (7 points) Evaluate $\displaystyle\int \frac{2}{x^2-2x-8}\,dx$.

A. $3\ln|x-4| - 3\ln|x+2| + C$

B. $\dfrac{2x}{\frac{x^3}{3} - x^2 - 8x} + C$

C. $\dfrac{1}{3}\ln|x-4| + \dfrac{1}{3}\ln|x+2| + C$

D. $\dfrac{1}{3}\ln|x-4| - \dfrac{1}{3}\ln|x+2| + C$

E. $\dfrac{2}{3}\tan^{-1}\left(\frac{x-1}{3}\right) + C$

10. (7 points) Evaluate $\displaystyle\int_2^\infty e^{-5x}\,dx$.

A. $\frac{1}{5}e^{-10}$

B. 0

C. $-\infty$

D. $5e^{-10}$

E. $-5e^{-10}$

11. (7 points) Find $f'(x)$ if $f(x) = x^{\sqrt{x}}$.

A. $\dfrac{\ln x}{2\sqrt{x}} + \dfrac{1}{\sqrt{x}}$

B. $x^{\sqrt{x}}\left(\dfrac{1}{2\sqrt{x}} \cdot \dfrac{1}{x}\right)$

C. $\sqrt{x} \cdot x^{\sqrt{x}-1}$

D. $x^{\sqrt{x}}\left(\dfrac{\ln x}{2\sqrt{x}} + \dfrac{1}{\sqrt{x}}\right)$

E. $x^{\sqrt{x}}(\sqrt{x}\ln x)$

12. (7 points) Find $f'(x)$ if $f(x) = \tan^{-1}(e^x)$.

A. $f'(x) = \dfrac{1}{1+e^{2x}}$

B. $f'(x) = -1\tan^{-2}(e^x)\cdot\sec^2(e^x)\cdot e^x$

C. $f'(x) = \dfrac{e^x}{\sqrt{1-e^{2x}}}$

D. $f'(x) = \dfrac{1}{\sqrt{1-e^{2x}}}$

E. $f'(x) = \dfrac{e^x}{1+e^{2x}}$

13. (7 points) If $f(x) = 2^{\cos x}$, find $f'(x)$.

A. $f'(x) = 2^{\sin x}$

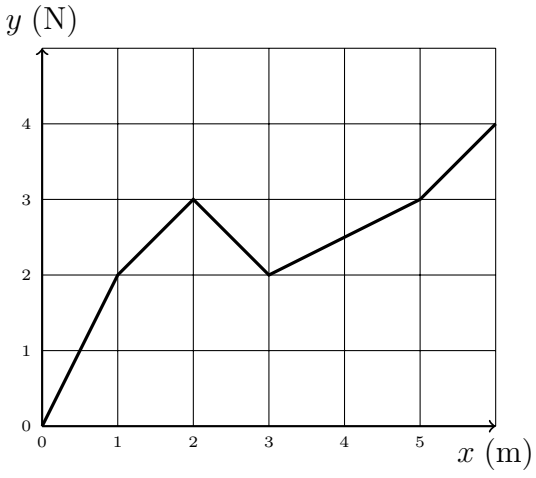
B. $f'(x) = -2^{\cos x}\cdot\sin x$

C. $f'(x) = -\sin x\cdot\ln 2\cdot 2^{\cos x}$

D. $f'(x) = \ln 2\cdot 2^{\cos x}$

E. $f'(x) = \dfrac{2^{1+\cos x}}{\cos x}$

14. (7 points) To the right is the graph of a force function (in Newtons). How much work is done by the force in moving an object from $x = 1$ meter to $x = 5$ meters?



A. 5 N·m

B. 6 N·m

C. 10 N·m

D. 11 N·m

E. 14.5 N·m

Congratulations you are now done with the exam!
Go back and check your solutions for accuracy and clarity. Make sure your final answers are BOXED.

When you are completely happy with your work please bring your exam to the front to be handed in.
Please have your MSU student ID ready so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
3	18	
4	18	
5	18	
6	18	
7	18	
8	21	
9	21	
10	21	
Total:	153	

No more than 150 points may be earned on the exam.