READ THE FOLLOWING INSTRUCTIONS.

- Do not open your exam until told to do so.

- No calculators, cell phones or any other electronic devices can be used on this exam.

- Clear your desk of everything except pens, pencils and erasers.

- If you need scratch paper, use the back of the previous page.

- Without fully opening the exam, check that you have pages 1 through 9.

- Fill in your name, etc. on this first page.

- **Show all your work.** Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don’t skip limits or equal signs, etc. Include words to clarify your reasoning.

- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.

- If you have any questions please raise your hand and a proctor will come to you.

- There is no talking allowed during the exam.

- You will be given exactly 90 minutes for this exam.

I have read and understand the above instructions: ____________________________

SIGNATURE
1. (5 points) Which of the following are considered elliptical paraboloids? *Circle all that apply.*
   A. $4x^2 = 9y^2 + z^2$
   B. $4x^2 + 9y^2 + z^2 = 36$
   C. $36z = 4x^2 + 9y^2$
   D. $36z = 4x^2 - 9y^2$
   E. $z = 4x^2 + 9y^2 + 36$
   F. None of the above

2. (5 points) The contour plot to the right could be from which function?
   A. $z = x^2 - y^2$
   B. $z = y(1 + x^2)$
   C. $z = \frac{x}{x^2 + 2y}$
   D. $z = \frac{1}{1 + x^2 + y^2}$
   E. $z = (x + 1)^2 + y^2$
   F. None of the above

Extra Work Space.
3. (8 points) Find the velocity and position functions of a particle that satisfies the following conditions.

\[ a(t) = \cos t \mathbf{i} + 6t \mathbf{j} + 4e^{2t} \mathbf{k} \]
\[ \mathbf{v}(0) = \mathbf{j} + 2 \mathbf{k} \]
\[ \mathbf{r}(0) = \mathbf{i} + 4 \mathbf{k} \]

(a) \( \mathbf{v}(t) = \) ________________________________

(b) \( \mathbf{r}(t) = \) ________________________________

4. (10 points) Let \( \mathbf{A} = 2 \mathbf{i} - 3 \mathbf{j} \) and \( \mathbf{B} = \mathbf{i} + 6 \mathbf{j} + 2 \mathbf{k} \). Answer the questions below.

(a) \( 3\mathbf{A} - 4\mathbf{B} = \) ________________________________

(b) Let \( \theta \) be the angle between \( \mathbf{A} \) and \( \mathbf{B} \). Then \( \cos \theta = \) ________________________________

5. (6 points) A parametrization of the line segment from \((2, -3, 1)\) to \((3, 3, -5)\) can be given by

\[ \mathbf{r}(t) = \) ________________________________ \(, \ t \in [0, 1]. \]
6. (12 points) Find the length of the curve below.

\[ r(t) = \frac{3t^2}{2} \mathbf{i} - 2t^{3/2} \mathbf{j} + \frac{3t}{2} \mathbf{k}, \quad 1 \leq t \leq 4 \]

7. Let \( f(x, y) = \ln(16 - 2x^2 - 4y^2) \).

(a) (8 points) Sketch the domain of \( f \).

(b) (4 points) Give the range of \( f \). Express your answer using interval notation.
8. (16 points) Find the following limits or show that they do not exist.

(a) \[ \lim_{(x,y) \to (0,0)} \frac{5xy}{x^2 + 2xy} \]

(b) \[ \lim_{(x,y) \to (0,0)} \frac{\cos x}{x - 2 \cos y} \]

9. Let \( f(x, y) = 3xy - x^2 \ln y \).

(a) (12 points) Find the linearization of \( f(x, y) \) at the point \((2, 1)\).

(b) (6 points) Use the linearization from part (a) to approximate \( f(1.9, 1.2) \).
10. (12 points) Find the equation of the plane containing the points $A$, $B$, and $C$ if

$$A = (1, 0, 0), \quad B = (3, -1, -1), \quad C = (-2, 3, 1)$$

11. Let $w = x^2z + ye^z$ with $x = st$, $y = t^2$, and $z = \ln s$.

(a) (8 points) Use the Chain Rule (from Calculus 3) to find $\frac{\partial w}{\partial s}$. No credit for any other method.

(b) (4 points) Find $\left.\frac{\partial w}{\partial s}\right|_{(s,t)=(1,1)}$
12. (10 points) Let \( f(x, y) = 8x^3y - 3x^2y^2 \). Find \( f_{xx} \) and \( f_{yy} \).

\[
\begin{align*}
f_{xx} &= 48xy - 6y^2 \\
f_{yy} &= -6x^2
\end{align*}
\]

13. (10 points) Find a vector function that represents the curve of the intersection of the surfaces below.

The paraboloid: \( z = 9 - x^2 - 3y^2 \)

The cylinder: \( z = y^2 \)

\[
\begin{align*}
x(t) &= \quad \quad \\
y(t) &= \quad \quad \\
z(t) &= t^2
\end{align*}
\]
14. (14 points) Let $T$ be the plane tangent to surface $z = x^2 + 2xy$ at the point $P(1, 0, 1)$. Find the distance from the plane $T$ and the point $Q(5, 3, -1)$. 
Congratulations you are now done with the exam! Go back and check your solutions for accuracy and clarity. Make sure your final answers are BOXED.

When you are completely happy with your work please bring your exam to the front to be handed in. Please have your MSU student ID ready so that is can be checked.

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