Michigan State University
Department of Mathematics

Name: ____ PID: _________________ Section: ________

Page	Max. Point Score	Actual Point Score
2	22	
3	33	
4	40	
5	30	
6	45	
7	30	
Total	200	

Please read the following directions carefully.

1. **DO NOT OPEN THIS EXAM BOOKLET UNTIL YOU ARE INSTRUCTED TO DO SO.**
2. Write your full name, your P. I. D. and your Math 124 section number at the top of this page, then finish reading these directions and ***sign the bottom of this page.***
3. WITHOUT FULLY OPENING THE EXAM, check the page numbers in this exam booklet. Including this cover page, you should have 7 *different* pages. If you do not, please request another copy immediately.
4. Neither books nor scratch paper are needed for this exam. Clear your desk of everything but this booklet, your pencils and your calculator. If you need more space to write your solutions, use the backs of the exam pages.
5. Crib sheets (pre-compiled lists of formulas or other information) either written or in a calculator are specifically forbidden. ***Use of a crib sheet of any kind on this exam will result in an automatic zero grade.***
6. The problems on this exam vary in difficulty. You should try to solve these problems in an order that will maximize your score: Solve all the easier problems first, then go back to the ones that require more thought.
7. Unless otherwise indicated, SHOW ALL YOUR WORK. If no work is shown, no partial credit can be awarded. *Even for calculator solutions, you should include relevant information, like the equation to be solved, the function whose graph is to be sketched, etc.*
8. Please place your answers in the boxes provided.
9. Unless you are specifically instructed to do otherwise, DO NOT ROUND YOUR ANSWERS!

I have read and fully understand all of the above instructions.

Signature: ____

1. (2+5+10+5=22) The following table gives the population of a small town in twenty-year increments between the January 1, 1880 and January 1, 2000:

Year	1880	1900	1920	1940	1960	1980	2000
Pop.	118	188	273	415	590	868	1252

Be as accurate as possible in calculating your answers to the following questions, and show your work carefully!

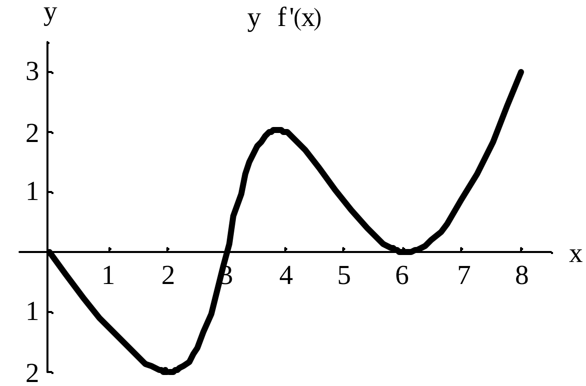
a) Find the change in the population between 1900 and 1940. ***Include the correct units.***

b) Find the *average* rate of change of the population between 1880 and 1940. ***Include the correct units.***

c) Estimate the *instantaneous* rate of change of the population on January 1, 1980. SHOW YOUR WORK. ***Include the correct units.***

d) *Within the context of this problem*, explain the practical meaning of the value that you found in part (c).

2. (2+2+2+2+2+3+3+2=18 points) The sketch on the right contains the *entire* graph of f' (***not*** f). Use this sketch to ***CAREFULLY*** answer each of the following questions:



a) The graph of the function f is concave __________ (up? down?) for $0 < x < 2$.

b) For $4 < x < 6$, the function f is _________________ (increasing? decreasing?).

c) The *point(s) of inflection* on the graph of f are located at $x =$ ______________________ (list ***all*** of them).

d) **Circle the larger value:** $f(1)$ $\quad$ $f(4)$.

e) There is a *local minimum* on the graph of the function f at $x =$ ______.

f) **Circle the *one* correct choice:** At $x = 6$, there is

   i) A local minimum for *f*.

   ii) A local maximum for *f*.

   iii) Neither a local maximum nor a local minimum for *f*.

g) **Circle the *one* correct choice:** At $x = 4$, there is

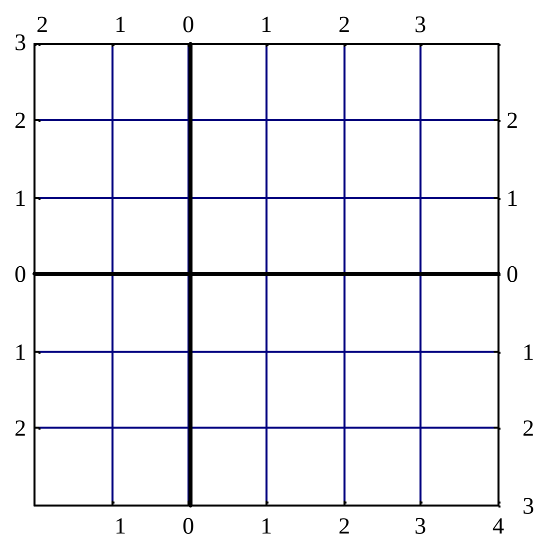
   i) A local maximum of f'.

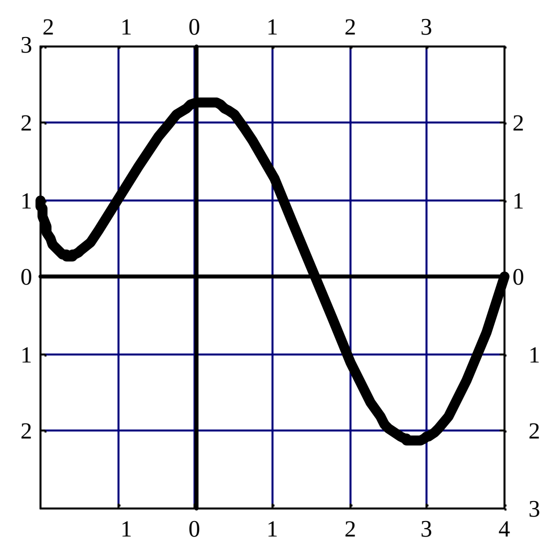
   ii) A point of inflection of f'.

   iii) A local maximum of f''.

h) The one ***positive*** value for c which makes $f(c) = f(0)$ is ________________ (give your best estimate).

3. (15 points) The graph of the function f is sketched in the box on the right. Sketch the corresponding graph of f' in the empty grid provided below.





4. (5+10+10=25 points) In what follows, $f(x)=5+3x-x^2$. ***Grading is based upon method. Credit is available only for answers that include complete and correct supporting work!***

a) Find $\dfrac{f(x+h)-f(x)}{h}$ and simplify it as much as possible. (**Notice: you should NOT find a limit here!**)

b) Evaluate $\lim\limits_{h\to 0}\dfrac{f(3+h)-f(3)}{h}$ exactly.

c) Find the equation of the line that is tangent to the graph of f at $x=3$.

5. (5+5+5=15 points) Short answer questions. Very little partial credit is available.

a) If $G(x)=\int_1^x \dfrac{\ln(t+1)}{t}\,dt$, then $G'(x)=$ ____________________?

b) If $\int_{-1}^{6} g(t)\,dt=-9$ and $\int_{-1}^{3} g(t)\,dt=-3$, then $\int_{3}^{6} g(t)\,dt=$ _________________________.

c) From the following table of values for F (***not*** F'), we see that $\int_1^3 F'(t)\,dt$ = ______________________.

t	0	1	2	3
F(t)	5	3	1	-1

6. (9+6+6+9=30 points) Evaluate each of the following. *You need not simplify your answers.*

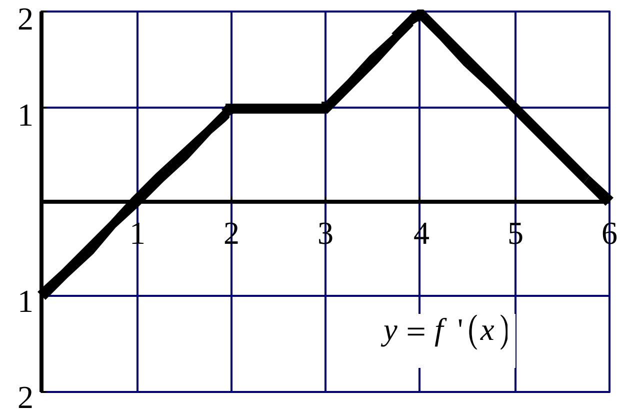
a) Find $f'(x)$ for $f(x)=\sin(2x)\cdot\cos(3x)$.

b) Find $\frac{dw}{dz}$ for $w=2\ln(3z)-3z\ln(2)$.

c) Find the velocity $v(t)$ from the position $s(t)=\frac{e^{3t}-5}{e^{2t}+1}$.

d) Find the marginal cost $C'(q)$ for the cost function $C(q)=2^q+\sqrt{2}+\sqrt{q}-q^2$.

7. (5+10+10=25 points) Use the graph of the function f' (***not*** f) that is sketched on the right to answer the following questions.



a) At x = 1, the graph of f is concave ________ (up? down?).

b) If $f(0) = -2$, then $f(3) =$ ____________________.

c) The **total change** in the value of f (***not*** f') between $x = 0$ and $x = 6$ is ____________________. SHOW YOUR WORK!

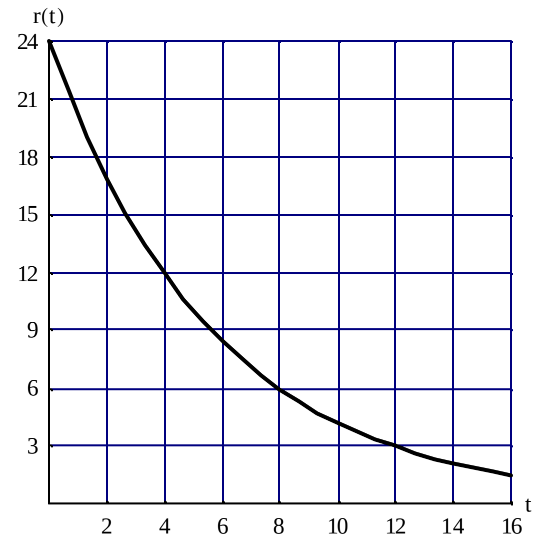
8. (5+10+5 = 20 points) $C(q)$ is the total cost (in dollars) required to set up a new factory and produce q units of a product. The *fixed* costs in $C(q)$ are \$30,000, and the *marginal* cost is $C'(q) = -0.004q^2 + 5q + 100$. In its first month of operation, your new factory produced 200 units.

a) Write a *definite integral* whose value is the ***change in the cost*** (that is, the ***variable cost***) for the first 200 units. *(Write only the correct integral! Numeric answers will receive no credit.)*

b) Find the ***total cost*** of everything required to produce the first 200 units. ROUND YOUR ANSWER TO THE NEAREST *DOLLAR*. (HINT: Use your calculator's built-in integration function.)

c) Find the ***price per unit*** which you must charge to break even at the end of your first month of production. ROUND YOUR ANSWER TO THE NEAREST *PENNY*.

9. (6+6+6+6+6=30 points) The graph of $r(t)$, on the right, shows the rate (in owls per hour) at which owls leave a forest filled with people trying to count them.



a) On the sketch, ***carefully*** shade in ***only*** the precise region whose area is given by the **right hand sum** with four subdivisions ($n = 4$) for $\int_0^{16} r(t)\, dt$.

b) What are the correct units for $\int_0^{16} r(t)\, dt$?

c) In the context of this problem, what is the practical meaning of $\int_0^{16} r(t)\, dt$?

d) WORKING CAREFULLY, evaluate the **right hand sum** with four subdivisions ($n = 4$) for $\int_0^{16} r(t)\, dt$.

e) Is the answer to part d) larger or smaller than the true value of $\int_0^{16} r(t)\, dt$?