MTH 126 Name: ______________

Spring Semester 2001 PID: ______________

Final Exam

Monday, Apr. 30 2001, 10.00 am - 12.00 noon *Signature*: ______________

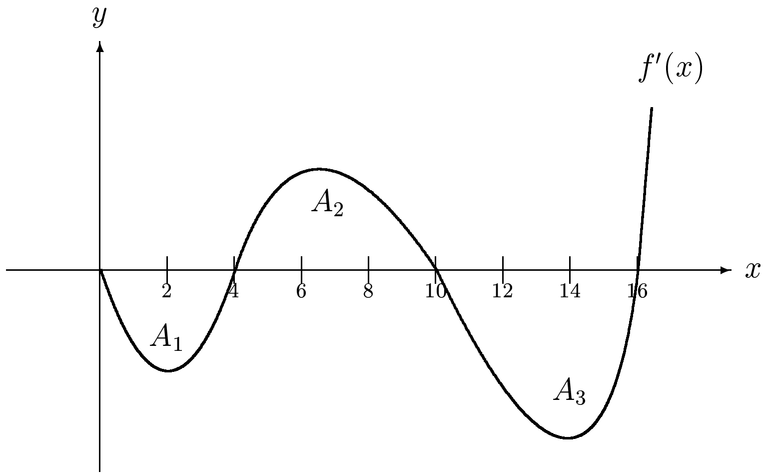
Page	1	2	3	4	5	6	7	8	9	10	Total
Score											/200

Show your work! No credit will be given for any answer without correct supporting work. For nay answer based on a graph from your calculator, sketch the graph and explain how it was used to obtain the answer.

1. (a) (9 points) Solve the indefinite integral $\int (e^{2x} - \frac{2}{\sqrt{x}} + \frac{1}{2x})\, dx$.

(b) (6 points) Solve the definite integral $\int_1^4 (e^{2x} - \frac{2}{\sqrt{x}} + \frac{1}{2x})\, dx$ using the Fundamental Theorem of Calculus. Show all your work. You are not supposed to use the $fnInt$, $\int$, or *int* feature on your calculators.

2. The graph of the derivative $f'(x)$ of a function $f(x)$ is given below.



Suppose you know that $f(4) = 14$, area $A_1 = 3$, area $A_2 = 6$ and area $A_3 = 11$.

(a) (5 points) Find the value of $f(16)$.

(b) (5 points) Find the value of $f(0)$.

For part (c) consider x just on the interval $0 \leq x \leq 16$.

(c) (5 points) On which interval(s) is f increasing? On which interval(s) is f decreasing?

(d) (5 points) What is the average value of f' on the interval $4 \leq x \leq 16$?

3. Let

$$p(x) = \begin{cases} 0 & x < 0 \\ 0.021e^{-0.021x} & x \geq 0 \end{cases}$$

be the density function for the annual precipitation (in inches) in Michigan.

(a) (5 points) What is the probability of having between 10 and 40 inches of precipitation in any given year?

(b) (5 points) What is the probability of having less than 10 inches of precipitation in any given year?

(c) (5 points) What is the probability of having more than 40 inches of precipitation in any given year?

(d) (5 points) Find the formula for the cumulative distribution function $P(x)$.

(e) (5 points) What is the median annual precipitation?

4. Suppose the yield z (in pounds) of tomatoes grown in a greenhouse depends on the amount of water x (in gallons) and the amount of fertilizer y (in kilograms) used. This means $z = f(x, y)$. Suppose that

$$f(x, y) = 20 \cdot y - (x - 10)^2 + 60 \qquad 0 \le x \le 20, \quad 0 \le y \le 5$$

(a) (4 Points) Write a formula for the function $f(5, y)$ and use it to fill in the following table:

y	0	1	2	3	4	5
$f(5, y)$						

(b) (4 Points) Write a formula for the function $f(x, 2)$ and use it to fill in the following table:

x	0	5	10	15	20
$f(x, 2)$					

(c) (6 Points) What is the value of $f_y(6, 1)$? What are the units of $f_y(6, 1)$? What is the interpretation of the value you found for $f_y(6, 1)$?

(d) (6 Points) Which of the following represents the Contour Diagram of $z = f(x, y)$?

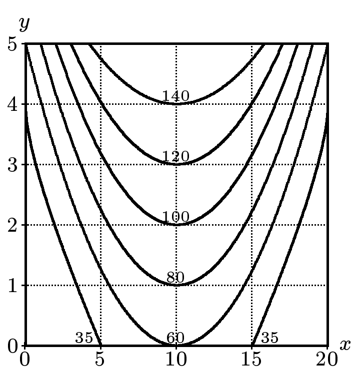


Diagram A

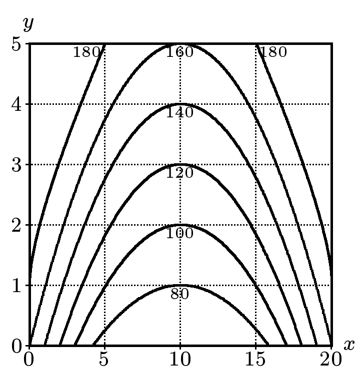


Diagram B

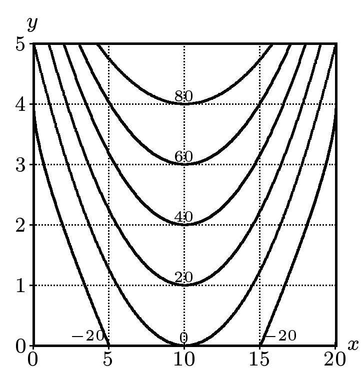


Diagram C

5. Let $f(x) = y^3 - 3xy + 6x$.

(a) (8 points) Find the critical points of $f(x, y)$.

(b) (10 points) Use the Second Derivative Test to determine whether each critical point is a local minimum, a local maximum or neither.

6. (10 points) Consider the solution of the differential equation $y' = y - 3x$ passing through $y(1) = 1$.Use Euler's method with step size $\Delta x = 0.2$ to estimate the solution at $x = 1.2,\ 1.4,\ 1.6$.

7. Consider the function $f(x, y) = \ln(x^2 y) + \sin x + 2$.

(a) (5 points) Find the formula for $f_x(x, y)$.

(b) (6 points) Find the formula for $f_y(x, y)$.

(c) (5 points) Find the formula for $f_{yy}(x, y)$.

8. The following table gives the values of a function $f(x, y)$:

	$x = 0$	$x = 10$	$x = 20$	$x = 30$
$y = 0$	89	80	74	71
$y = 2$	93	85	80	76
$y = 4$	98	91	85	81
$y = 6$	104	98	92	88
$y = 8$	112	105	99	94

(a) (5 points) Estimate $f_x(10, 6)$ and $f_y(10, 6)$.

(b) (5 points) Use the above partial derivatives to estimate $f(12, 5)$.

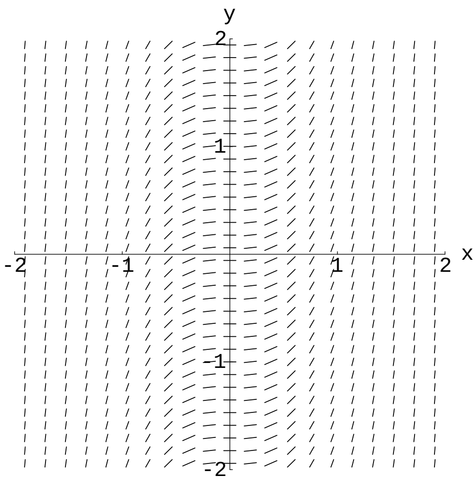
9. Find the general solution of the following differential equations.

(a) (7 points) $y' = y^2 + (yx)^2$

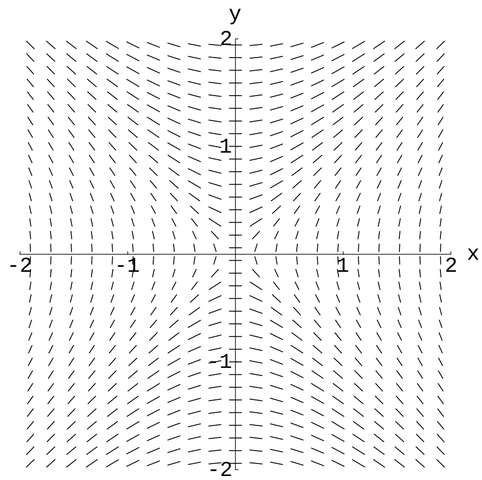
(b) (6 points) $y' = ye^x - y$

(c) (6 points) $\dfrac{d^2 s}{dt^2} + 4s = 0$

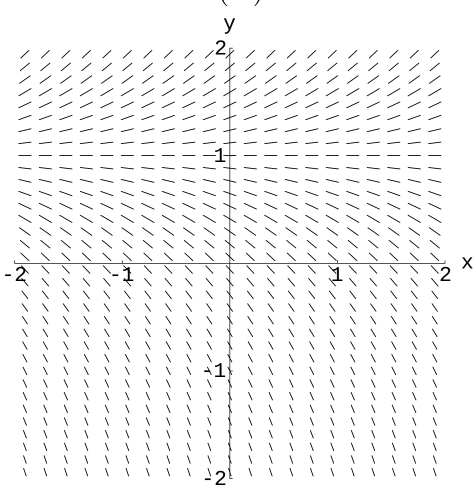
10. (9 points) Below are given four slope fields.



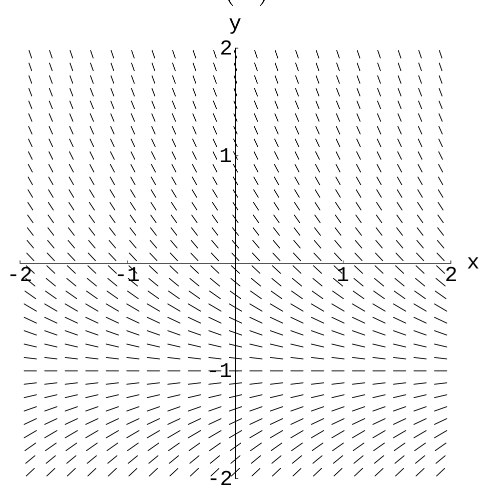
(A)



(B)



(C)



(D)

Match the following differential equations with their slope fields.

________ $y' = y - 1$

________ $y' = \dfrac{x}{y}$

________ $y' = -1 - y$

________ $y' = 3x^2$

11. You heat up a cup of water in your microwave, put in some tea and leave it to cool on your desk.

(a) (10 points) If you know that the room temperature is 70^oF, write down and solve a differential equation describing the temperature $H(t)$ of your tea, t minutes after you took it out of the microwave.

(b) (10 points) Suppose you know in addition that the temperature of the water was 170^oF when you took it out of the microwave and had cooled down to 150^oF in 5 minutes. If you like to take your tea at 130^oF, how long do you have to wait?

12. You want to be able to predict how pollution of a certain lake changes over time. You know that pollutants enter the lake at a constant rate of 300 lb/year and that the amount leaving the lake through its outlets is proportional to the quantity that is already there, with the constant of proportionallity 0.8. Denote by $Q(t)$ the total amount of pollutants in the lake (measured in pounds), t years after you started monitoring the lake.

(a) (7 points) Write a differential equation for $Q(t)$.

(b) (7 points) If you know that initally there were 4000 lb of pollutant in the lake, solve the differential equation from (a).

(c) (4 points) In the long run, what will be the total amount of pollutants in this lake?