Topics

In this document we will review

- Finding the domain of a rational function.
- Finding the zeros of a rational function.
- Solving rational inequalities
- **Note:** You should look of the "Manipulation of Fractions" document to review simplifying rational expressions.

Why do we need this?

Examples are listed below from several different sections and chapters.

Example 1 (From Section 3.3)**.** Find the intervals on which $\dfrac{x}{x^2+1}$ is increasing or decreasing.

Example 2 (From Section 3.3)**.** Find the intervals on which $\dfrac{x}{x^2+1}$ is concave up and concave down.
List any inflection points.

Example 3 (From Section 3.5)**.** Sketch the curve $\dfrac{x}{x^3-1}$

Example 4 (From Section 3.5)**.** Sketch the curve $1+\dfrac{1}{x}+\dfrac{1}{x^2}$

Important Definitions and Theorems

Remark 5. Remember how to factor and the difference of squares
(gone over in the "Manipulation of Fractions" review sheet)

Theorem 6. Take $R(x)=\dfrac{p(x)}{q(x)}$ for polynomials $p(x)$ and $q(x)$.
Then $x=a$ is not in the domain of $R(x)$ if and only if $q(a)=0$.

Theorem 7. If $R(x)=\dfrac{p(x)}{q(x)}$ for polynomials $p(x)$ and $q(x)$ then
$R(a)=0$ if and only if a is in the domain of $R(x)$ and $p(a)=0$.

Theorem 8 (Strategy for solving rational inequalities)**.**

1. Get 0 on one side of the inequality and your rational function (we will call $R(x)$) on the other side.
2. Plot the solutions to the equation $R(x)=0$ on a number line.
3. Plot values of x that are not in the domain of $R(x)$.
4. This has split the number line into several intervals. In each interval take a value a and look at the sign of $R(a)$.
   - If $R(a)>0$ then $R(x)$ is positive on the entire interval.
   - If $R(a)<0$ then $R(x)$ is negative on the entire interval.
5. Check your endpoints and put it all together.

Instructional Videos

Click on the following links to access helpful instructional videos:

- Solving rational inequalities
  `www.math.msu.edu/classes/mth_132/review_video/rat2.aspx`
- Solving rational inequalities (more advanced)
  `www.math.msu.edu/classes/mth_132/review_video/rat3.aspx`

Now that you have been exposed to all the ideas and seen a couple solutions worked out you should try a few problems. Please see the quiz which has some questions for you to try and the answers posted at the end. The important thing is the work that leads to the answers. That's where you come in!