Most physical phenomena are described by more than one dependent variables. In class we have just encountered the case of parametrized curves where to describe the position of an object, as a function of time, we need to simultaneously examine its x-coordinate and y-coordinate as a function of time.

Today we will explore parametrized curves, and start you thinking about how to apply the calculus concepts of integrals and derivatives to these curves, ahead of the rigorous discussion we will give in class. We will start our discussion with the help of the Etch A Sketch toy, and finish by analyzing the vehicle data from a 1963 Corvette Grand Sport on the Laguna Seca race track.

**Etch a Sketch**

The Etch a Sketch is a toy invented in the 1960s.

![Etch a Sketch toy](https://commons.wikimedia.org/wiki/File:Taj_Mahal_drawing_on_an_Etch-A-Sketch.jpg) used under CC-by-SA license.

The toy has two knobs. Turning the left one moves a tracer up and down, and turning the right one moves a tracer left and right. Mathematically, this means that the vertical rate of change of the tracer is given by the rate at which you twist the left knob, and the horizontal rate of change of the tracer is given by the rate at which you twist the right knob.

The following graphic illustration will walk you through a stimulation of the Etch a Sketch toy using some simple MATLAB commands to prepare you for more complicated examples in the lab.
Left knob

Let imagine the tracer starting at the point $(1, 1)$, to plot this on MATLAB, we would use this line of code

```
plot(1,1,'ro')
```

and as you can see, the tracer, a red circle, is now located at the point $(1, 1)$.

Now we will turn the left knob steadily at 2 units per second. We started at the point $(1, 1)$, but after turning the knob, the tracer moves up 2 units, so it ends at the point $(1, 3)$. We plot the line and the tracer with the following code
The plot command first takes the list of $x$ values and then the list of $y$ values.

**Right knob**

We now turn the right knob steadily at $-1$ unit per second, for a length of 1 second. At the end of the last step, the tracer was at the point $(1, 3)$. Next we turned the right knob, so the tracer moved to the left by 1 unit, ending at the point $(0, 3)$. We record all of our movement so far in the following line of code producing the corresponding trace on the Etch a Sketch.
Two knobs together

We now turn the left knob and the right knob together, with the left knob going at -1 units per second, and the right knob moving at 2 units per second, for a duration of 1 second.

The left knob controls the $y$-coordinate, so moving at $-1$ units per second for 1 second will decrease the $y$-coordinate by 1. Starting from the point $(0,3)$, it will move to a point with $y$-coordinate 2. As the right knob controls the $x$-coordinate, moving it at 2 units per second for 1 second will increase the $x$-coordinate by 2. Starting from $(0,3)$, it will move to a point with $x$-coordinate 2. To trace the path with the final coordinate as $(2,2)$, we use the following code.

```python
plot([1,1,0,2],[1,3,3,2],'r')
plot(2, 2, 'ro')
```

How far has the tracer traveled?

There are three steps to this process.

- In the first step, the tracer moved up by 2 units.
- In the second step, the tracer moved sideways by 1 unit.
- In the final step, the tracer moved diagonally. It moved 2 units right and 1 unit down. The distance traveled in this step can be found using the Pythagorean theorem: the distance traveled is the length of the hypotenuse. So,

$$
\text{distance} = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{2^2 + 1^2} = \sqrt{1 + 4} = \sqrt{5}
$$

So in total, the distance traveled by the tracer is $2 + 1 + \sqrt{5} = 3 + \sqrt{5}$ units.