1. Let \( n \) be a positive integer. Prove that the sum of the digits of \( 1981^n \) is at least 19.

2. Let \( A \) be a square matrix with real entries such that \( A^3 = A + I \). Show that \( \det A > 0 \).

3. You have a chess board of size \( 10 \times 10 \). Prove that there are no more than \( 2^{50} \) ways to tile the board with dominoes (i.e., rectangles of size \( 2 \times 2 \)).

4. Let \( f \) be a real continuous function on \( [-1, 1] \) such that \( |f(t)| \leq 1 \) for \( t \in [-1, 1] \), \( \int_{-1}^{1} f(t) \, dt = 1 \), and \( \int_{-1}^{1} f^2(t) \, dt = 1 \). Show that
   (a) \( \int_{-1}^{1} f^3(t) \, dt \geq 0 \);
   (b) \( \int_{-1}^{1} f^3(t) \, dt \geq 1/3 \).

5. A part of a square of size \( 1 \times 1 \) is painted in red. It is known that no two red points of the square are at the distance \( \varepsilon \) (where \( \varepsilon > 0 \)). Prove that
   (a) the area of the red part of the square is at most \( \frac{1}{3}(1 + \varepsilon)^2 \).
   (b) the area of the red part of the square is at most \( \frac{2}{7}(1 + \sqrt{3}\varepsilon)^2 \).