The Seventh Annual Herzog Prize Examination

November 3, 1979

Problem 1: (D. Moran) Find N, given that N is a prime which can be written as an alternating sequence of 0's and 1's.

Problem 2: (L. Kelly) Let $f(x)$ be a continuous real-valued function mapping $[0,1]$ into $[0,1]$ with $f(0) = 0$. If for every $x \in [0,1]$ there is a positive integer n such that

$$f^{(n)}(x) = x \quad \text{(n-th iterate)}$$

show that $f(x) = x$ for all x.
(Note: $f^{(2)}(x) = f(f(x))$, $f^{(3)}(x) = f(f(f(x)))$ etc.).

Problem 3: (J. Kinney) Let A,B and C be the vertices of an equilateral triangle of side 1. Let R be the intersection of the closed discs of radius 1 with centers A,B and C. Can R be covered by two closed discs of diameter 1?

Problem 4: (A. Baily) Recall that $e^{\pi i} = -1$. Justify

$$e^{\pi\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}} = -\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$$

Problem 5: (A.M.M. E 2791) Suppose $\sum_1^\infty a_n$ converges. Must $\sum_1^\infty a_n^3$ converge?

Problem 6: (L. Kelly) A square matrix has the property that the interchange of any two distinct rows produces a symmetric matrix. Characterize such matrices.