The 29th Herzog Competition

Problem 1. Let p and q be two consecutive odd primes (like 5 and 7 or 23 and 29). Prove that $p + q$ can be represented as a product of 3 integer factors, each of the factors being greater than 1.

Problem 2. An n by n square filled with numbers $1, 2, \ldots, n^2$ is called magic if the sum of the numbers in each row, each column, and each of the two diagonals is the same. Does there exists a magic 4 by 4 square that contains numbers 2, 3, and 4 in the positions shown below?

2	3		
4			

Problem 3. During the last 11 weeks John played at least one game of chess every day but not more than 12 games during any week. Prove that there was a period of several consecutive days during which John played exactly 22 games.

Problem 4. Find (with proof) all mappings F from $\mathbb{Z}_{239}$ (the ring of residues modulo 239) into itself satisfying

$$F(x + y) = F(x)F(y) \quad \text{for all } x, y \in \mathbb{Z}_{239}.$$

Problem 5. Prove that for every $x > 0$,

$$e^{-x} + e^{-1/x} \geq \frac{2}{e}.$$