Project Description

The project focuses on the study of self-assembly of bilayer structures in multicomponent amphiphilic mixtures (Figure 0.1), which are abundant in biological structures, such as cell membranes, and have wide-range applications in pharmaceutics, emulsion stabilization, detergent production and energy conversion devices [1, 2, 3, 5, 4]. An experimental example is given in Figure 0.1, showing internal microphase separation of multicomponent pores, formed from blends of $PS_{9.5K}-PEO_{5K}$ (grey) with $PS_{56K}-P2VP_{21K}$ (dark), into pearled P2VP cores (a & b) and cylindrical pores (c), [6].

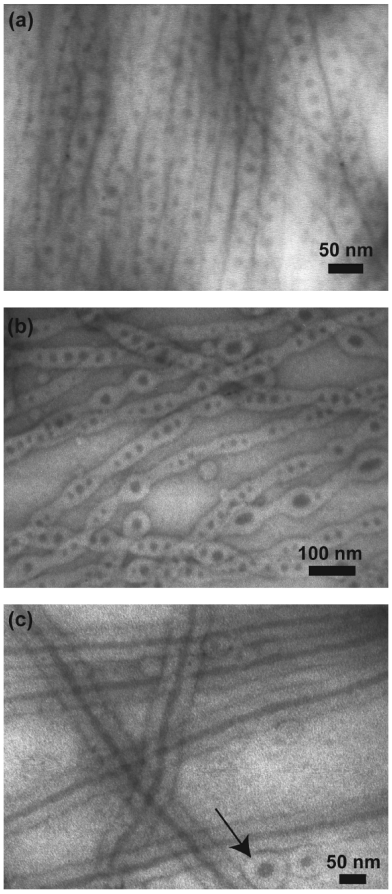

Figure 0.1: TEM images of multicompartment pores

Mathematically we use a combination of elementary differential geometry and simple dynamical systems ideas to construct competitors for the minima of a free energy. We will present a local coordinate system (Left, Figure 0.2) which reduces the study of the minima to a simple second order dynamical system of the form

$$\begin{pmatrix} u_1 \\ \\ u_2 \end{pmatrix}_{xx} = \begin{pmatrix} \dfrac{\partial W}{\partial u_1}(u_1,u_2) \\ \\ \dfrac{\partial W}{\partial u_2}(u_1,u_2) \end{pmatrix},$$

where $\mathbf{u}=(u_1,u_2)\in\mathbb{R}^2$ and $W:\mathbb{R}^2\mapsto\mathbb{R}$ is a scalar valued potential. We are interested in the existence of homoclinic orbits of this system connecting to zero. A starting point is to consider the case when the potential profile is a billiard limit, shown as in the right of Figure 0.2.

Anyone who are interested in this project are welcome to contact us via qwu@math.msu.edu. Prerequisites include MTH 234-235 while MTH 340 and 441 are helpful but not essential.

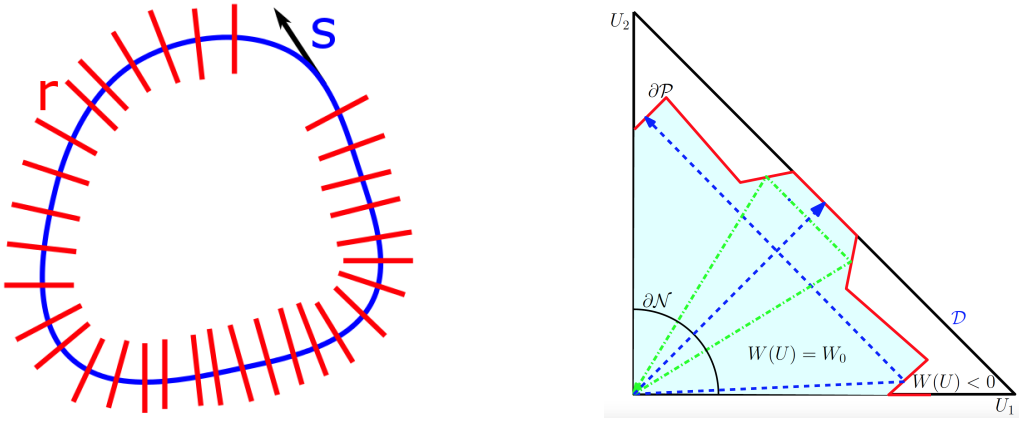

Figure 0.2: (Left) Local coordinates. (Right)A billiard limit of the potential W.

References

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