1.3 Quadratic Equations

In this section you will learn to:

- solve quadratics equations by
  1. factoring
  2. square root property
  3. quadratic formula
  4. completing the square
  5. graphing (used mainly for checking – not considered an algebraic solution)
- use the discriminant to find the number and type of solutions (roots, $x$-intercepts, zeros)

A quadratic equation in $x$ is an equation that can be written in the general form

$$ax^2 + bx + c = 0,$$

where $a$, $b$, and $c$ are real numbers, with $a \neq 0$. A quadratic equation in $x$ is also called a second-degree polynomial equation.

**The Zero-Product Principle:** If the product of two algebraic expressions is zero, then at least one of the factors equal to zero.

If $AB = 0$, then $A = 0$ or $B = 0$.

**Solving Quadratic Equations by Factoring:**

**Example 1:** $9x^2 = 12x$

**Example 2:** $x^2 = 3x + 10$

**Steps:**

1. Rearrange equation so that one side is 0. (Look for GCF.)
2. Factor. (Use sum/product idea when $a = 1$. If $a \neq 1$, use grouping*.)
3. Set each factor equal to 0.
4. Solve each equation.
5. Check in original equation or by graphing (observe $x$-intercepts).

*Refer to “Class Pages” on math home page (www.msu.edu) for steps using “Grouping Method”.
Example 3: \(4x^2 - 13x = -3\) (Use “guess & check” or “grouping method”.)

Solving Quadratic Equations by the Square Root Method:

**Square Root Property:** If \(a > 0\) then \(x^2 = a\) has two real roots: \(x = \sqrt{a}\) or \(x = -\sqrt{a}\)

Reminder: Any time you choose to take a square root when solving an equation, you must include \(\pm\).
(Example: If \(x^2 = 4\), then \(x = \pm 2\).)

Example 4: \(3x^2 - 1 = 47\)  
Example 5: \((8x - 3)^2 = 5\)

Solving Quadratic Equations Using the Quadratic Formula:

**Quadratic Formula:** If \(ax^2 + bx + c = 0\), where \(a \neq 0\), then

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

Example 6: Solve and simplify: \(3x^2 = 5x - 1\)
Example 7: Solve and simplify: \(4x^2 + 16x = 13\)

Example 8 (optional): Solve for \(t\) and simplify: \(h = 32t - 16t^2\)

In the quadratic formula, \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\), the value of \(b^2 - 4ac\) is called the discriminant.

Beware: The discriminant is NOT \(\sqrt{b^2 - 4ac}!!\)

<table>
<thead>
<tr>
<th>(b^2 - 4ac &gt; 0)</th>
<th>(b^2 - 4ac &lt; 0)</th>
<th>(b^2 - 4ac = 0)</th>
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Example 9: Determine the number and type of solutions for the equations below. (Do not solve.)

(a) \(2x - 3x^2 = 21\)  
(b) \(25x^2 + 49 = 70x\)
Solving a Quadratic Equation by Completing the Square:

Recall:  Perfect Square Trinomials

\[
(x + 3)^2 = x^2 + 6x + 9 \\
x^2 + 12x + 36 = (x + 6)^2
\]

\[
x^2 - 8x + ____ = (____)^2 \\
x^2 + 18x + ____ = (____)^2 \\
x^2 - ____ + 100 = (____)^2
\]

Example 10:  \[2x^2 + 5x - 3 = 0\]  

Steps:

1. Divide each term by the coefficient of \(x^2\).
2. Move the constant term to the right side.
3. “Form” a perfect square trinomial by adding \(1/2\) of the new coefficient of \(x\) to both sides.
4. Factor the left side (perfect square trinomial). Add the terms on the right side.
5. Finish solving using the Square Root Method.
1.3 Homework Problems

Solve Problems 1-6 by factoring:

1. $3x^2 = 5x$
2. $x^2 - 15 = 2x$
3. $-10x = x^2 + 25$
4. $a(a - 12) - 15 = 30$
5. $2x^2 - 4x = 30$
6. $3m^2 = 7m + 6$

Solve Problems 7-9 by using the quadratic formula:

7. $x^2 + 15 = 8x$
8. $4x^2 - 8x + 1 = 0$
9. $4x^2 = 2x + 7$

Solve Problems 10-12 using the square root method:

10. $x^2 = \frac{25}{49}$
11. $(2x + 11)^2 + 5 = 3$
12. $(3x - 4)^2 = 8$

13. Solve for $t$: $h = 16t^2 - 4$
14. Solve for $x$: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

For Problems 15-17, determine the number and type of solutions by examining the discriminant.

15. $-3x^2 = 21 - 2x$
16. $2x^2 - 20x + 49 = 0$
17. $9x^2 + 49 = 42x$

Solve each of the following quadratic equations by completing the square:

18. $x^2 + 8x + 15 = 0$
19. $x^2 - 3x = 10$
20. $4x^2 = 7 - 8x$

1.3 Homework Answers:  
1. \( \left\{ \frac{5}{3} \right\} \)  
2. \( \{-3, 5\} \)  
3. \(-5\)  
4. \(-3, 15\)  
5. \(-3, 5\)  
6. \(\left\{-\frac{2}{3}, 3\right\}\)

7. \(\{3, 5\}\)  
8. \(\left\{\frac{2 + \sqrt{3}}{2}\right\}\)  
9. \(\left\{\frac{1 + \sqrt{29}}{4}\right\}\)  
10. \(\left\{\pm \frac{5}{7}\right\}\)  
11. \(\phi\)  
12. \(\left\{\frac{4 + 2\sqrt{2}}{3}\right\}\)  
13. \(\left\{\pm \frac{\sqrt{h + 4}}{4}\right\}\)

14. \(\left\{\pm \frac{a\sqrt{b^2 + y^2}}{b}\right\}\)  
15. 0 real roots  
16. 2 real roots  
17. 1 real root  
18. \(\{-5, -3\}\)  
19. \(\{-2, 5\}\)  
20. \(\left\{-\frac{2 + \sqrt{11}}{2}\right\}\)