1.7 Inequalities

In this section you will learn to:

- use interval notation
- understand properties of inequality
- solve linear (and compound) inequalities
- solve polynomial inequalities
- solve rational inequalities

**Interval Notation:**

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Graph</th>
<th>Interval Notation</th>
<th>Set Builder Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &lt; 4 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x \geq -3 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-3 &lt; x \leq 2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x \leq -3 \text{ or } x &gt; 5 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all real #’s</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Intersection**

**Union**
### Properties of Inequalities

<table>
<thead>
<tr>
<th>+/- Property of Inequality</th>
<th>Multiplication/Division Property of Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td>If ( a &lt; b ), then</td>
<td></td>
</tr>
<tr>
<td>( a + c &lt; b + c )</td>
<td></td>
</tr>
<tr>
<td>( a - c &lt; b - c )</td>
<td></td>
</tr>
<tr>
<td>(Adding or subtracting does not affect the &gt; or &lt; sign.)</td>
<td></td>
</tr>
<tr>
<td>If ( a &lt; b ), then</td>
<td></td>
</tr>
<tr>
<td>( c &gt; 0 ) (( c ) is positive)</td>
<td>( ac &lt; bc )</td>
</tr>
<tr>
<td>( a &lt; \frac{b}{c} ) ( c &gt; 0 ) (( c ) is negative)</td>
<td>( ac &gt; bc )</td>
</tr>
</tbody>
</table>

(When multiplying or dividing by a negative number, reverse the > or < sign.)

---

**Example 1:** Solve and graph the inequality below. Write the answer using interval notation.

\[
2 - 3x \leq 5
\]

**Example 2:** Solve and graph the inequality below. Write the answer using interval notation.

\[
5 - (7x + 19) > 13(x - 5)
\]

**Example 3:** Solve the compound (“and”) inequality \(-3 < \frac{2}{3}x + 1 \leq 5\). Write answer using interval notation.

(a) Solve by isolating the variable \(x\).

(b) Solve by writing each inequality separately.

**Example 4:** Avis charges $40/day plus $.10/mile to rent a car. Hertz charges $50/day plus $.08/mile. When is Avis a better deal if you are renting a car for three days?
A polynomial inequality is any inequality of the form:

\[ f(x) < 0 \] (graph is below the \( x \)-axis)
\[ f(x) \leq 0 \] (graph is on or below the \( x \)-axis)
\[ f(x) > 0 \] (graph is above the \( x \)-axis)
\[ f(x) \geq 0 \] (graph is on or above the \( x \)-axis)

where \( f \) is a polynomial function.

**Example 5:** Solve \( x^2 - 6 > 5x \). Write the solution using interval notation.

**Steps for Solving Polynomial Inequalities:**

1. Express as \( f(x) > 0 \) or \( f(x) < 0 \).
   (Get 0 on right side.)
2. Set \( f(x) = 0 \) and solve for \( x \) to get **Boundary Points**.
3. Plot the boundary points on a number line to obtain **Intervals**.
4. **Test Values** within each interval and evaluate \( f(x) \) for each value.
   - If \( f(x) > 0 \), then \( f(x) \) is + for interval.
   - If \( f(x) < 0 \), then \( f(x) \) is – for interval.
5. Write the solution using interval notation.
   Check the solution on your calculator.

**Example 6:** Solve \( x^3 + x^2 - 9 \leq 9x \). Write the solution using interval notation.
Example 7: Solve $x^3 + 2x^2 \geq 4x + 8$. Write the solution using interval notation.

A rational inequality is any inequality of the form:

- $f(x) < 0$ (graph is below the x-axis)
- $f(x) \leq 0$ (graph is on or below the x-axis)
- $f(x) > 0$ (graph is above the x-axis)
- $f(x) \geq 0$ (graph is on or above the x-axis)

where $f$ is a rational function. ($f(x) = \frac{p(x)}{q(x)}$, where $p$ and $q$ are polynomials and $q(x) \neq 0$)

Example 8: Solve $\frac{x-2}{x+5} \geq 0$. Write the solution using interval notation.

Steps for Solving Rational Inequalities:

1. Express as $f(x) > 0$ or $f(x) < 0$.  
   (Get 0 on right side.)

*2. Find values that make the numerator & the denominator = 0. These are the Boundary Points. (Note Restrictions!)

3. Plot the boundary points on a number line to obtain Intervals.

4. Test Value within each interval and evaluate $f(x)$ for each value.
   If $f(x) > 0$, then $f(x)$ is + for interval.
   If $f(x) < 0$, then $f(x)$ is – for interval.

5. Write the solution using interval notation.
   Check the solution on your calculator.
Example 9: Solve \( \frac{x}{x+2} \geq 2 \). Write the solution using interval notation.

Example 10: A ball is thrown vertically from a rooftop 240 feet high with an initial velocity of 64 feet per second. During which time period will the ball’s height exceed that of the rooftop? (Use \( h(t) = -16t^2 + v_0t + s_0 \) where \( v_0 \) = initial velocity, \( s_0 \) = initial height/position, and \( t \) = time. (You may also want to graph this function on your calculator using the viewing rectangle [0, 10, 1] by [-100, 500, 100]).
1.7 Homework Problems

Solve each of the inequalities below and write the answer using interval notation.

1. \(3 + 5x \leq 2(1 + 3x)\)
2. \(3(x + 2) \leq 4(x + 5)\)
3. \(\frac{3(x+3)}{2} \leq \frac{2(x + 7)}{3}\)

4. \(\frac{2}{3} x - x \leq -\frac{3}{2}(x - 5)\)
5. \(\frac{1}{4} x + \frac{2}{3} x - x > \frac{1}{2} + \frac{1}{2}(x + 1)\)
6. \(2 + x < 3x - 2 \leq 5x + 2\)

7. \(0 \leq \frac{3 + x}{2} < 4\)
8. \(x^2 - 2x - 8 > 0\)
9. \(2x^2 + x - 3 \leq 0\)

10. \(x^3 + x^2 \leq 4x + 4\)
11. \(x^3 \geq 9x^2\)
12. \(x^3 - 2x^2 - 4x + 8 \leq 0\)

13. \(9x^2 - 6x + 1 < 0\)
14. \(\frac{2}{x} < 4\)
15. \(\frac{x - 4}{x + 3} > 0\)

16. \(\frac{4 - 2x}{3x + 4} \leq 0\)
17. \(\frac{-x + 2}{x - 4} \geq 0\)
18. \(\frac{x + 1}{x + 3} \leq 2\)

19. \(\frac{3}{x - 2} \geq 5\)
20. \(\frac{x}{2x - 1} - 1 \geq 0\)

1.7 Homework Answers:

1. \([-1, \infty)\)
2. \([-14, \infty)\)
3. \((-\infty, \frac{1}{5})\)
4. \((-\infty, \frac{45}{7})\)
5. \((-\infty, -\frac{12}{7})\)

6. \((2, \infty)\)
7. \([-3, 5)\)
8. \((-\infty, -2) \cup (4, \infty)\)
9. \([-\frac{3}{2}, 1]\)
10. \((-\infty, -2] \cup [-1, 2]\)
11. \([0] \cup [9, \infty)\)
12. \((-\infty, -2] \cup \{2\}\)
13. \(\emptyset\)
14. \((-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)\)
15. \((-\infty, -3) \cup (4, \infty)\)
16. \((-\infty, -\frac{4}{3}) \cup [2, \infty)\)
17. \([2, 4]\)
18. \((-\infty, -5] \cup (-3, \infty)\)
19. \(\left(2, \frac{13}{5}\right]\)
20. \(\left(\frac{1}{2}, 1\right]\)