

Any of the following exercises are fair game for your final oral exam. I suggest that you write up your solutions neatly in your own handwriting to consult during that exam! All assigned book exercises are from the textbook available here:

<https://users.math.msu.edu/users/iwenmark/Teaching/MTH994/Fall12022/HDP-book.pdf>.

PROBLEMS ASSIGNED FROM CHAPTERS 5, AND FROM DISCUSSION OF FAST JL

33. Do exercise 5.1.2 from the book.
34. Do exercise 5.1.3 from the book.
35. Do exercise 5.1.8 from the book.
36. Do exercise 5.1.9 from the book.
37. Do exercise 5.1.12 from the book.
38. Do exercise 5.1.15 from the book.
39. Do exercise 5.2.4 from the book.
40. Prove Khintchine's Inequality: Let X_1, \dots, X_n be independent sub-gaussian random variables with zero means. Let $\mathbf{a} \in \mathbb{R}^n$ and $K := \max_j \|X_j\|_{\psi_2}$. Then $\forall p \geq 1$, $\|\sum_{j=1}^n a_j X_j\|_{L^p} \leq CK\sqrt{p} \|\mathbf{a}\|_2$.
Hint: Everything you need can be found in sections 2.5 and 2.6 of the book. In particular, this is a slight (effectively free) generalization of the upper bound portion of book exercise 2.6.5 (assigned as HW problem 9).
41. Let $H \in \mathbb{R}^{n \times n}$ be such that $H^T H = nI_n$, and $D \in \{0, 1\}^n$ be a random diagonal matrix with ± 1 's on its diagonal. Show that $\|HD\mathbf{x}\|_2 = \sqrt{n}\|\mathbf{x}\|_2$ holds for all $\mathbf{x} \in \mathbb{R}^n$ almost surely (i.e., with probability 1). Note: The probability distribution on the signs on the diagonal of D doesn't matter.