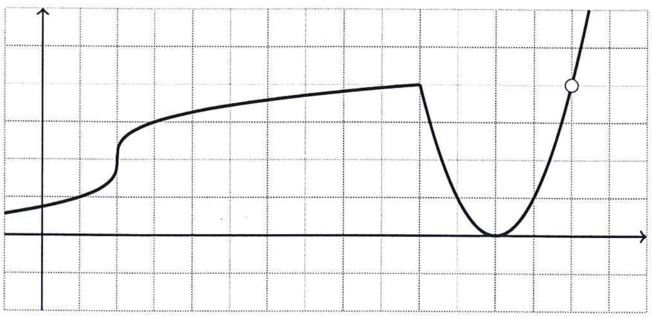
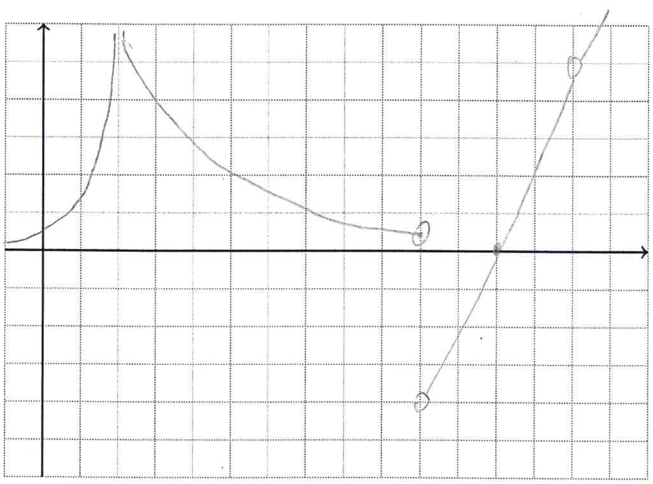
2.2 Problems

COMMENTS FOR INSTRUCTORS: The vertical tangent will create a vertical asymptote on the derivative graph.

The Derivative

Example 1. Sketch the curve for the derivative of the function shown.





Example 2. Write the definition of the derivative of a function from memory.

Example 3.

(a) If $f(x) = 2x^2 - 13x + 5$, use the definition of the derivative to find $f'(x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2 - 13(x+h) + 5 - (2x^2 - 13x + 5)}{h}$$

$$= \lim_{h\to 0} \frac{2h^2 + 4xh - 13h}{h}$$

$$= \lim_{h\to 0} 2h + 4x - 13$$

$$= 4x - 13$$

(b) Use the result of part a to find the equation of the tangent line at the point where x=3.

$f(3) = 2\cdot 3^2 - 13\cdot 3 + 5 = -16$

The tangent line at $(3, -16)$ is

$y + 16 = f'(3)(x-3)$

Use (a), $f'(3) = 4\cdot 3 - 13 = -1$, plugging it to above

$y + 16 = (-1)(x-3) \iff y = -x - 12$

Example 4. If $f(x) = \dfrac{1}{\sqrt{x}}$ find $\dfrac{d}{dx}f(x)$

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \lim_{h\to 0} \frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h} \cdot \frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}$$

$$= \lim_{h\to 0} \frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$$

$$= \lim_{h\to 0} \frac{-h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$$

$$= \frac{-1}{\sqrt{x}\sqrt{x}(2\sqrt{x})} = -\frac{1}{2}x^{-3/2}$$

COMMENTS FOR INSTRUCTORS: Many students may already know the answer to this from physics, but it serves to reinforce the real world application of 2nd derivative.

Example 5. If the position of a particle is given by $h(t) = -16t^2 + 96t + 10$ use the definition of the derivative to find the acceleration function.

$a(t)$: acceleration function

$a(t) = h''(t)$

Take the derivative twice.

First time: $h'(t) = \lim_{\Delta t \to 0} \frac{h(t+\Delta t) - h(t)}{\Delta t}$

$= \lim_{\Delta t \to 0} \frac{-16(t+\Delta t)^2 + 96(t+\Delta t) + 10 - (-16t^2 + 96t + 10)}{\Delta t}$

$= \lim_{\Delta t \to 0} \frac{-16\Delta t^2 - 32t\Delta t + 96\Delta t}{\Delta t}$

$= \lim_{\Delta t \to 0} -16\Delta t - 32t + 96$

$= -32t + 96$

Second time $h''(t) = \lim_{\Delta t \to 0} \frac{h'(t+\Delta t) - h'(t)}{\Delta t}$

$= \lim_{\Delta t \to 0} \frac{-32(t+\Delta t) + 96 - (-32t + 96)}{\Delta t}$

$= \lim_{\Delta t \to 0} \frac{-32\Delta t}{\Delta t}$

$= -32$

Hence $a(t) = h''(t) = -32$