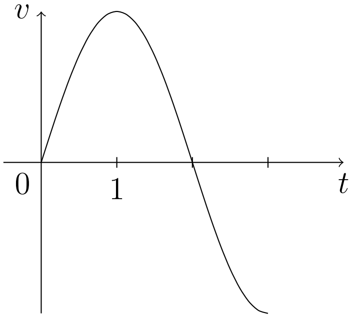
2.7 Problems

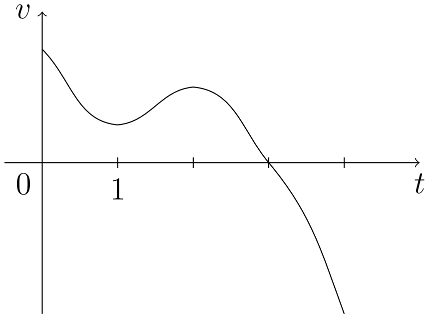
Graphs

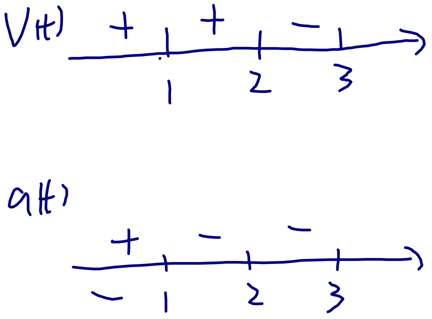
Example 1. Graphs of the velocity functions of two particles are shown, where t is measured in seconds. When is each particle speeding up? When is it slowing down? Explain.

(a)



(b)



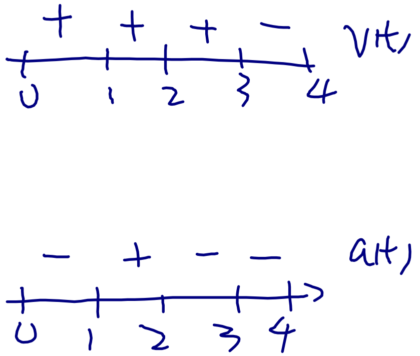


speeding up:

$(0,1)\cup(2,3)$

slowing down

$(1,2)$



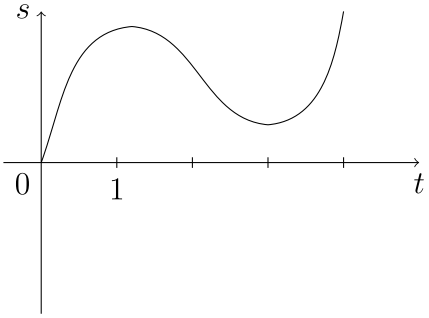
up: $(1,2)\cup(3,4)$

down:

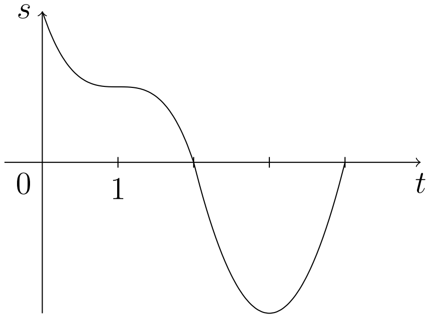
$(0,1)\cup(2,3)$

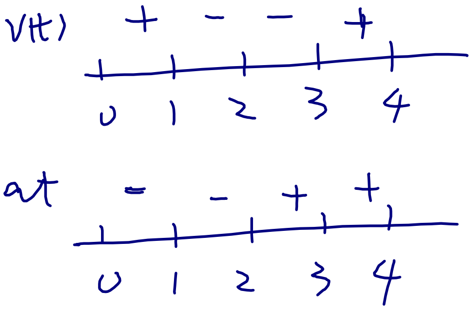
Example 2. Graphs of the position functions of two particles are shown, where t is measured in seconds. When is each particle speeding up? When is it slowing down? Explain.

(a)



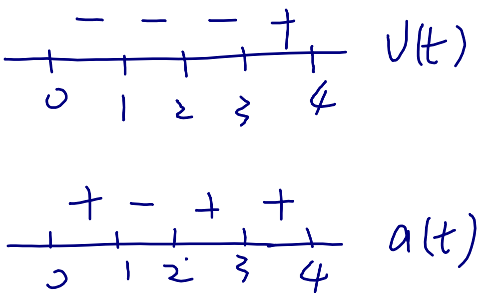
(b)





up $(1,2)\cup(3,4)$

down: $(0,1)\cup(2,3)$



up $(1,2)\cup(3,4)$

down $(0,1)\cup(2,3)$

Standard Problems

Example 3. A particle moves according to the position function $s(t) = \dfrac{t}{(1+t^2)}$ on the interval $t \geq 0$, where t is measured in seconds and s in feet.

(a) Find the velocity at time t.

$$v(t) = s'(t) = \frac{1\cdot(1+t^2) - t\cdot 2t}{(1+t^2)^2}$$

(b) When is the particle at rest.

at rest means $v(t) = 0$

$$\Leftrightarrow \frac{1+t^2 - t\cdot 2t}{(1+t^2)^2} = 0 \Rightarrow 1+t^2 - t\cdot 2t = 0$$

$$\Rightarrow t^2 = 1 \Rightarrow t = 1, \boxed{-1}$$

X

↑

as $t \geq 0$

$\Rightarrow t = 1$ seconds

(c) When is the particle moving in the positive direction?

moving to positive direction means $v'(t) > 0$

$$\Leftrightarrow \frac{1+t^2 - t\cdot 2t}{(1+t^2)^2} > 0$$ solve inequality

$\Leftrightarrow 1 - t^2 > 0$

($1 - t^2 = 0 \Rightarrow t = \pm 1$

$-\quad +\quad -$ $\quad 1-t^2$

$-1 \quad 1$)

$\Leftrightarrow t \in (-1, 1)$ }

Recall $t \geq 0$ }

$\Rightarrow t \in [0, 1)$ s

(d) Find the total distance traveled in first 8 seconds

The turning points are where $v'(t) = 0$

from (b) we know it happens at $t = 1$

So Total distance

$$= |s(1) - s(0)| + |s(8) - s(1)|$$

$$= \left|\frac{1}{2} - 0\right| + \left|\frac{8}{1+8^2} - \frac{1}{2}\right| = \frac{1}{2} + \frac{1}{2} - \frac{8}{65} = 1 - \frac{8}{65} \text{ feet}$$

(**Example 3** continued) Recall $s(t) = \dfrac{t}{(1+t^2)}$ on the interval $t \geq 0$.

(e) Calculate the acceleration of the particle at time t.

recall $v(t) = \dfrac{1-t^2}{(1+t^2)^2}$

$$a(t) = v'(t) = \frac{-2t(1+t^2)^2 - (1-t^2)\cdot 2(1+t^2)\cdot 2t}{(1+t^2)^4}$$

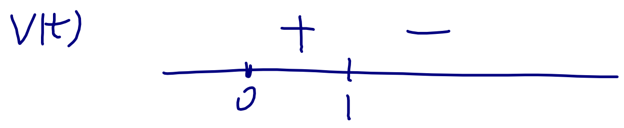
$$= \frac{-2t(1+t^2) - 4t(1-t^2)}{(1+t^2)^3}$$

$$= \frac{-2t-2t^3-4t+4t^3}{(1+t^2)^3} = \frac{-6t+2t^3}{(1+t^2)^3}$$

(f) When is the particle speeding up?

speeding up means $v(t)>0$ & $a(t)>0$ or
$v(t)<0$ & $a(t)<0$

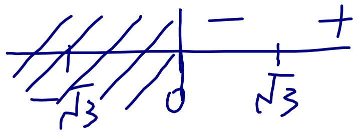
From (c), we know $v(t)>0$ on $[0,1)$, so



We need to find when $a(t)>0$. From (e), this means

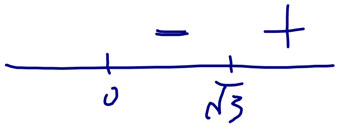
$\dfrac{-6t+2t^3}{(1+t^2)^3} > 0 \Leftrightarrow -6t+2t^3>0 \leftarrow$ Solve this

$-6t+2t^3=0 \Rightarrow t=0, \pm\sqrt{3}$



recall $t \geq 0$

so a(t)



speeding up on $(1,\sqrt{3})$

Example 4. A ball is thrown vertically upward on planet X with an initial velocity of 10 meters per second. Its height after t seconds is given by $h(t) = -at^2 + 10t + 1$

(a) Find the value of a if the ball reaches its maximum height after 5 seconds

It reaches maximum height at $h'(t)=0$

$\Rightarrow$ it reaches maximum height after 5 seconds means

$$h'(5)=0$$

$$\Leftrightarrow (-2at+10)\big|_{t=5}=0$$

$$\Leftrightarrow -10a+10=0 \Rightarrow a=1 \Rightarrow h(t)=-t^2+10t+1$$

(b) What is the balls maximum height?

by (a), maximum height is the height at $t=5$, so it is

$$h(5) = -5^2+10\cdot 5+1$$

$$= -25+50+1$$

$$= 26$$

(c) When will the ball hit the ground?

hit the ground means $h(t)=0$, solve for t

$$\Leftrightarrow -t^2+10t+1=0$$

$$t=\frac{10\pm\sqrt{10^2+4}}{2}=5\pm\sqrt{26}$$

since $t\geq 0$, it can only be $5+\sqrt{26}$

(d) How fast is the ball traveling when it is 2 meters above the ground on the way down?

$$v(t)=h'(t)=-2at+10$$

when $h(t)=2 \Leftrightarrow -t^2+10t+1=2$

$$\Leftrightarrow t^2-10t+1=0$$

$$\Leftrightarrow t=\frac{10\pm\sqrt{100-4}}{2}=5\pm\sqrt{24}$$

$t=5-\sqrt{24}$ is on the way up (due to (a))

$\boxed{t=5+\sqrt{24}}$ is on the way down, at this time $v(t)=-2(5+\sqrt{24})+10$

$$=-2\sqrt{24}$$