2.9 Problems

Theorem 1. $\frac{d}{dx}[\sin(x^\circ)] \neq \cos(x^\circ)$

Proof.

$$\frac{d}{dx}(\sin x^\circ) = \frac{d}{dx}\sin\left(\frac{\pi}{180}x\right) = \frac{\pi}{180}\cos\left(\frac{\pi}{180}x\right) = \frac{\pi}{180}\cos(x^\circ) \neq \cos(x^\circ)$$

□

Example 2. Use linear approximation and differentials to approximate $\sin(33^\circ)$ by following the steps below.

(a) Find the linearization of $\sin x$ at the appropriate $x = a$.

$a = 30^\circ = \frac{\pi}{6}$ $\quad f(x) = \sin x \quad x_0 = \frac{\pi}{180} \cdot 33$

The linearization $L(x) = f(a) + f'(a)(x-a)$

$$= \sin\frac{\pi}{6} + \cos\frac{\pi}{6}\left(x - \frac{\pi}{6}\right)$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$$

(b) Use the linearization to approximate $\sin(33°)$.

$$\sin 33^\circ = \sin\left(\frac{33\pi}{180}\right) \approx L(x_0) \overset{\text{by (a)}}{=} \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x_0 - \frac{\pi}{6}\right)$$

$\uparrow$ x_0

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{33\pi}{180} - \frac{\pi}{6}\right)$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{3\pi}{180}$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{120}\pi$$

(c) Find the differential of $y = \sin x$.

$$dy = y'\,dx$$
$$= \cos x\,dx$$

(d) Use the appropriate dx to evaluate the differential for dy. What does this give you? Why is it different from your answer in (b)?

x changes from $\frac{\pi}{6}$ to $\frac{33\pi}{180}$, so $\boxed{dx = \frac{33\pi}{180} - \frac{\pi}{6} = \frac{3\pi}{180}}$

by (c) $\quad dy = \cos\frac{\pi}{6} \cdot dx$

$$= \frac{\sqrt{3}}{2} \cdot \frac{3\pi}{180}$$

$$= \frac{\sqrt{3}\,\pi}{120}$$

$\leftarrow$ It is just the second term in the answer to (b)