Appendix E Problems

Sigma Notation

Example 1. Prove the formula for the sum of the first n positive integers.

Example 2. Find m and n such that $9 + 27 + 81 + 243 = \sum_{i=m}^{n} 3^i$

$3^2 + 3^3 + 3^4 + 3^5$

$= \sum_{i=2}^{5} 3^i$

Example 3. Write the sum in sigma notation:

(a) $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36}$

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} = \sum_{i=1}^{6} \frac{1}{i^2}$

(b) $3 - 8 + 15 - 24 + 35 - 48$

$(2^2-1) - (3^2-1) + (4^2-1) - (5^2-1) + (6^2-1) - (7^2-1)$

$= \sum_{i=2}^{7} (i^2-1)(-1)^i$

Example 4. Find the number n such that $\sum_{i=1}^{n} i = 78$

$$\frac{n(n+1)}{2} = 78$$

$$\underbrace{n}_{\uparrow \text{ integer}}(n+1) = 2\cdot 2\cdot 39 = \underbrace{2\cdot 2\cdot 3}_{12}\cdot 13 = 12\cdot 13$$

$$n = 12$$

Example 5. Find the value of the sum.

(a) $\sum_{k=0}^{92} \cos k\pi$ $= \cos 0 + \cos\pi + \cos 2\pi + \cos 3\pi + \cdots + \cos 92\pi$

$$(1 \quad -1) \quad (1 \quad -1) \quad (1 \quad -1) \quad 1$$

$$= 1$$

(b) $\sum_{j=1}^{n}(j+1)(j+2)$ $= \sum_{j=1}^{n} (j^2 + 3j + 2)$

$$= \sum_{j=1}^{n} j^2 + 3\sum_{j=1}^{n} j + 2n$$

$$= \frac{n(n+1)(2n+1)}{6} + 3\cdot\frac{n(n+1)}{2} + 2n$$

(c) $\sum_{i=1}^{n} i(4i-3)$ $= \sum_{i=1}^{n} 4i^2 - 3i = 4\sum_{i=1}^{n} i^2 - 3\sum_{i=1}^{n} i$

$$= \frac{4n(n+1)(2n+1)}{6} - 3\frac{n(n+1)}{2}$$

(d) $\sum_{i=5}^{25} -3i^2$ $= \sum_{i=5}^{25} (-3i^2) = -3\sum_{i=5}^{25} i^2$

$$= -3\left(\sum_{i=1}^{25} i^2 - \sum_{i=1}^{4} i^2\right)$$

$$= -3\left(\frac{25\cdot 26\cdot 51}{6} - \frac{4\cdot 5\cdot 9}{6}\right)$$